

ON WAVE MOTION IN AN INFINITE SOLID BOUNDED INTERNALLY BY A CYLINDER OR A SPHERE

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PART I

In two previous papers,† the author investigated the problem of wave motion for infinite domains of one, two, and three dimensions and for certain sub-infinite domains; that is, domains bounded in certain directions but extending to infinity in other directions. The present paper is a sequel to the aforementioned papers and deals with the problem of wave motion in an infinite solid, bounded internally by a cylinder or a sphere.

In the subsequent developments we shall use the following abbreviations:

$$\sigma(\alpha) = (a^2\alpha^2 - k^2)^{1/2}, \quad s(p, \alpha) = \alpha^2 + (p^2 - k^2)/a^2,$$

where α is a real variable ranging from $-\infty$ to ∞ and p is a complex variable whose real part is positive. We shall also introduce the operators ∇_c , ∇_s , $\sum \iiint$, and $\sum \iiint \iiint$ defined as follows:

$$\begin{aligned} \nabla_c &= \frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} - \frac{1}{a^2} (p^2 - k^2) + \frac{1}{r^2} \frac{\partial^2}{\partial \theta^2}, \\ \nabla_s &= \frac{\partial^2}{\partial r^2} + \frac{2}{r} \frac{\partial}{\partial r} + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2}{\partial \phi^2} \\ &\quad - \frac{1}{a^2} (p^2 - k^2), \end{aligned}$$

$$\begin{aligned} \sum \iiint \iiint \{F_n(r', \theta', \alpha)\} &= \sum_{n=0}^{\infty} (2n+1) \cos n(\theta - \theta') \int_R^{\infty} r' dr' \\ &\quad \cdot \int_0^{2\pi} d\theta' \int_{-\infty}^{\infty} \alpha F_n(r', \theta', \alpha) d\alpha, \end{aligned}$$

$$\begin{aligned} \sum \iiint \iiint \iiint \{F_n(r', \theta', \phi', \alpha)\} &= \sum_{n=0}^{\infty} (2n+1) P_n(\cos \gamma) \\ &\quad \cdot \int_R^{\infty} r'^{3/2} dr' \int_0^{\pi} \sin \theta' d\theta' \int_0^{2\pi} d\phi' \int_{-\infty}^{\infty} \alpha F_n(r', \theta', \phi', \alpha) d\alpha, \end{aligned}$$

† *On wave motion for infinite domains*, Philosophical Magazine, (7), vol. 26 (1938), pp. 340–360; *On wave motion for sub-infinite domains*, Philosophical Magazine, (7), vol. 27 (1939), pp. 182–194. These papers will be referred to as L-1 and L-2, respectively.