SOME INVARIANTS UNDER MONOTONE TRANSFORMATIONS*

D. W. HALL† AND A. D. WALLACE

We assume that S is a locally connected, connected, compact metric space and that P is a property of point sets. For any two points a and b of S we denote by $C(ab)$ (respectively $C_i(ab)$) a closed (closed irreducible) cutting of S between the points a and b. We consider the following properties:

$\Delta_0(P)$. *If S is the sum of two continua, their product has property P.*

$\Delta_1(P)$. *If K is a subcontinuum of S and R is a component of $S-K$, then the boundary of R, $(F(R)=\overline{R}-R)$, has property P.*

$\Delta_2(P)$. *Each $C_i(ab)$ has property P.*

$\Delta_3(P)$. *If A and B are disjoint closed sets containing the points a and b, respectively, there is a $C(ab)$ disjoint from $A+B$ and having property P.*

If P is the property of being connected, the four properties $\Delta_i(P)$ are equivalent as shown by Kuratowski.‡ Indeed it may be seen that Kuratowski's proofs allow us to state the following theorem:

THEOREM 1. *For any property P of point sets, $\Delta_i(P)$ implies $\Delta_{i+1}(P)$ for $i=0, 1, 2$.*

This result is the best possible in the sense that there is a property (that of being totally disconnected) for which no other implication holds.

The single-valued continuous transformation $T(S)=S'$ is said to be *monotone* if the inverse of every point is connected. It may be seen that the following statements are true:§

(i) *The inverse of every connected set is connected.*

(ii) *If the set X separates S between the inverses of the points x and y, then $T(X)$ separates S' between x and y.*

THEOREM 2. *If the property P is invariant under monotone trans-*

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† National Research Fellow.

‡ C. Kuratowski, *Une caractérisation topologique de la surface de la sphère*, Fundamenta Mathematicae, vol. 13 (1929), p. 307, and references given there.

§ G. T. Whyburn, *Non-alternating transformations*, American Journal of Mathematics, vol. 56 (1934), p. 294.