A NOTE ON THE ASYMPTOTIC PROPERTIES OF ORTHOGONAL POLYNOMIALS*

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Let $\psi(u)$ be a function non-decreasing in the interval (0, 1) such that all the moments

$$c_k = \int_0^1 u^k d\psi(u), \qquad k = 0, 1, 2, \cdots,$$

exist, and let c_0 be positive. Then

$$(1) \qquad f(z) \equiv \int_0^1 \frac{d\psi(u)}{z-u} = \sum_{r=0}^{\infty} \frac{c_r}{z^{r+1}}$$

may be developed in a continued fraction of which the denominators of the successive approximants are the Tchebichef polynomials $Q_n(z)$,† where

$$\Delta_n Q_n(z) = \begin{vmatrix} c_0 & c_1 & \cdots & c_n \\ c_1 & c_2 & \cdots & c_{n+1} \\ \cdot & \cdot & \cdots & \cdot \\ c_{n-1} & c_n & \cdots & c_{2n-1} \\ 1 & z & \cdots & z^n \end{vmatrix}, \qquad \Delta_n = \begin{vmatrix} c_0 & c_1 & \cdots & c_n \\ c_1 & c_2 & \cdots & c_{n+1} \\ \cdot & \cdot & \cdots & \cdot \\ c_{n-1} & c_n & \cdots & c_{2n-2} \end{vmatrix},$$

$$n = 0, 1, 2, \cdots; \Delta_0 = 1.$$

The determinants Δ_n are positive unless $\psi(u)$ has only a finite number ν of points of increase, in which case $\Delta_n = 0$ for $n > \nu$, and the continued fraction is terminating.

Shohat‡ has shown that for an extensive class of moment functions (1) we have

$$(2) \qquad (\Delta_n/\Delta_{n+1})^{1/2} \sim 4^n,$$

and that for all functions of this type satisfying (2) the recurrence

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† J. Shohat, *Théorie Générale des Polynomes Orthogonaux de Tchebichef*, Paris, 1934, p. 12.

‡ J. Chokhatte (Shohat), *Sur le développement de l'intégrale* $\int_a^b [p(y)/(x-y)]dy$ *en fraction continue et sur les polynomes de Tchebycheff*, Rendiconti del Circolo Matematico di Palermo, vol. 47 (1923), p. 32.