

each two nonparallel elements of G cross each other. Obviously the conclusions of the theorem do not hold.

The following example will show that the condition that no two elements of the collection G shall have a complementary domain in common is also necessary. In the cartesian plane let M be a circle of radius 1 and center at the origin, and N a circle of radius 1 and center at the point (5, 5). Let G_1 be a collection which contains each continuum which is the sum of M and a horizontal straight line interval of length 10 whose left-hand end point is on the circle M and which contains no point within M . Let G_2 be a collection which contains each continuum which is the sum of N and a vertical straight line interval of length 10 whose upper end point is on the circle N and which contains no point within N . Let $G = G_1 + G_2$. No element of G crosses any other element of G , but uncountably many have a complementary domain in common with some other element of the collection. However, it is evident that no countable subcollection of G covers the set of points each of which is common to two continua of the collection G .

It is not known whether or not the condition that each element of G shall separate some complementary domain of every other one can be omitted.

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A PRINCIPAL AXIS TRANSFORMATION FOR NON-HERMITIAN MATRICES

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The availability of the principal axis transformation for hermitian matrices often simplifies the proof of theorems concerning them. In working with non-hermitian matrices (square or rectangular) it was found that a generalization of this transformation has a similar use for them.* A special case of this generalization has been investigated by Sylvester† who proved Theorem 1 (below) for square matrices with real elements. The unitary matrices U and V are in that case orthogonal matrices with real elements. Special cases had also been

* C. Eckart, *The kinetic energy of polyatomic molecules*, Physical Review, vol. 46 (1934), p. 383; C. Eckart and G. Young, *The approximation of one matrix by another of lower rank*, Psychometrika, vol. 1 (1936), p. 211; A. S. Householder and G. Young, *Matrix approximation and latent roots*, American Mathematical Monthly, vol. 45 (1938), p. 302.

† Sylvester, Messenger of Mathematics, vol. 19 (1889), p. 42.