CONCERNING CONTINUA IN A SEPARABLE SPACE WHICH DO NOT CROSS*

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In working with collections of continua it is sometimes useful to know something of the character of the point set consisting of all the points common to two or more members of the collection. Also it is of advantage to know conditions under which we may subtract a countable number of continua from the collection and have left a collection of mutually exclusive continua. The theorem which we shall prove may aid in answering questions of this nature.

All definitions and discussions will refer to point sets in a connected and locally connected separable space S.

DEFINITION 1. *If g_1 and g_2 are two continua each of which separates S' g_1 will be said to cross g_2 provided there exist two complementary domains of g_2 (maximum connected domains of $S-g_2$) which contain points of g_1.*

DEFINITION 2. *If* (1) *M_1, M_2, and M_3 are three point sets in S,* (2) *g_1 and g_2 are two continua in S,* (3) *M_1 is in a complementary domain D_1 of g_1 which does not contain a point of g_2,* (4) *M_2 is in a complementary domain D_2 of g_2 which does not contain a point of g_1, and* (5) *M_3 is in a complementary domain D_3 of g_1+g_2 distinct from D_1 or D_2, then g_1+g_2 will be said to separate M_1, M_2, and M_3 symmetrically with respect to M .*

LEMMA 1. *If* (1) *g_1 and g_2 are two continua each of which separates S,* (2) *each separates some complementary domain of the other, and* (3) *neither crosses the other, then there exist domains D_1, D_2, and D_3 such that g_1+g_2 separates D_1, D_2, and D_3 symmetrically with respect to D_3.*

PROOF. $S-g_1$ is the sum of two mutually separated point sets S_1 and S_2. One of these, say S_1, is such that the continuum g_2 is a subset of S_1+g_1. Let D_1 be a maximum connected domain of S_2. Also $S-g_2$ is the sum of two mutually separated point sets S_3 and S_4. One of these, say S_3, is such that g_1 is a subset of S_3+g_2. Let D_2 be a maximum connected domain of S_4. Let D_3 be a maximum connected domain of g_1+g_2 distinct from D_1 or D_2. We know D_3 exists, since by hypotheses each of the continua g_1 and g_2 separates some complementary domain of the other. The domains D_1, D_2, and D_3 are then separated by g_1+g_2 symmetrically with respect to D_3.

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