ON FOURIER SERIES WITH RESTRICTED COEFFICIENTS*

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1. **Introduction.** Consider a real-valued function $f(x)$, periodic with period 2π and Lebesgue integrable. Let

$$f(x) \sim \frac{a_0}{2} + \sum_1^\infty (a_\nu \cos \nu x + b_\nu \sin \nu x) \tag{1.1}$$

be its Fourier series, and let $s_0 = a_0/2$,

$$s_n(f; x) = s_n = \frac{a_0}{2} + \sum_1^n (a_\nu \cos \nu x + b_\nu \sin \nu x), \quad n = 1, 2, 3, \cdots, \tag{1.2}$$

be its partial sums. We shall mainly restrict ourselves to series satisfying the conditions

$$na_n \geqq -p, \quad nb_n \geqq -p, \quad \text{for} \quad n = 1, 2, 3, \cdots, \tag{1.3}$$

where $p \geqq 0$. We shall in particular consider the following problem:

Suppose

$$-\mu \leqq f(x) \leqq \mu \quad \text{in} \quad -\pi < x < \pi; \tag{1.4}$$

then what is the best upper bound $C_n(\mu, p)$ for the partial sums $|s_n(f; x)| \leqq C_n(\mu, p)$, $(n \geqq 1)$, under the assumption (1.3)?

It is known that the sequence $\{C_n\}$ is bounded (cf. Szász [5], [6]);† hence l.u.b. $C_n(\mu, p) = C(\mu, p)$ is finite. For $p=0$ the author [9] proved recently that

$$C(\mu, 0) < (2 + 4/\pi)\mu < 3.3\mu; \tag{1.5}$$

for $p > 0$ the sharpest estimates so far were given by Fekete [2] using a device of Paley and Fejér [1]. Fekete proved that

$$C(\mu, p) < 5\mu + 6p, \tag{1.6}$$

and also that

$$C(\mu, p) < 5\mu + 8(\mu p)^{1/2}. \tag{1.7}$$

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† See the list of references at the end of this paper.