

THE APPLICABILITY OF LATTICE THEORY TO GROUP THEORY*

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The influence of a more general theory, such as the theory of lattices, upon a more specific theory, such as the theory of groups, may make itself felt in two directions: either one tries to generalize the known theorems of group theory in order to find their place in the more general framework of lattice theory, or else one tries to use the methods of lattice theory for the solution of problems in the theory of groups which this theory has not yet been able to solve by its own means. It is the latter kind of problem that interests us in this note.

But before attacking a problem like this, one may very well ask whether or not this problem has a solution, that is, whether we have any reason to expect that the methods of lattice theory may yield a solution of our problems, and this question of the applicability of lattice theory to group theory will occupy us in the course of this investigation.

The fundamental problem of group theory is the so-called structure problem. Here I use the term "structure" in the customary sense that two groups have the same structure if they are isomorphic. This problem has been solved for rather restricted classes of groups only, notably the finite abelian groups. In some of these cases it has actually been possible to rephrase the results in terms of lattice theory, whereas in other cases no such attempt has been made. The question is now whether group theory may expect from lattice theory the answer to this as yet unsolved problem, and it is a systematic attempt to find this out which gave rise to the following considerations.

In the specific case which we are discussing, we may state as our object a delineation of the extent to which the structure of a group is determined by the structure of its lattice of subgroups. The first observation one makes, as soon as one tries to put this on a more precise basis, is that of the wealth of obvious counterexamples which seem to indicate that, as far as our problem is concerned, we may not expect much help from lattice theory. Thus I noticed with much surprise that all these counterexamples are comparatively "small"

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