

THE REPRESENTATION OF BOOLEAN ALGEBRAS

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1. **Introduction.** In this brief address I shall set myself a twofold aim: to review the theory of Boolean algebras, as we understand it today, and to sketch certain historical phases of its development. Although I am in no sense prepared to offer the fruits of painstaking historical research, I believe that by a few pertinent historical observations I shall be able to bring out the underlying evolutionary pattern, which, in fact, is a quite familiar one.

2. **Motivation and essential features of the theory.** More than ninety years have passed since the publication of George Boole's first contribution towards an algebra of logic [5].* While Boole was by no means the first to attempt a symbolic method in logic (among his precursors we find Leibniz, Jacques and Jean Bernoulli, J. H. Lambert, and Gergonne [6]) it is a just and proper tribute to his genius that we commonly call this algebra by his name. I believe it would be accurate to say that of the many books, memoirs, notes, and reviews (more than one hundred seventy-five in number [6]) which deal with Boolean algebras the great majority draw their inspiration directly or indirectly from the work of Boole. The orientation of these studies toward symbolic logic is apparent in their preoccupation with algorithms, identities, and equations, or with the logical interrelations of the formal properties of the various Boolean operations. Recently there has emerged a different tendency, namely, to view Boolean algebras structurally, as organic systems, rather than algorithmically. Although this tendency might naturally have been expected to take its origin either in the rich experience of algebraists or in the needs of mathematicians concerned with the calculus of classes, it sprang, in fact, from quite different sources as a recognizable, if somewhat remote, consequence of the work of Hilbert. The most intensive exploitation of this new tendency is due to Tarski and myself [28]–[39]. Tarski's theory of deductive systems, which is but one illustration of the way in which logic has been enriched by the sort of metamathematical inquiry first seriously attempted by Hilbert, deals with systems of propositions which are complete with respect to logical inference; from a mathematical point of view, it is therefore a theory of the relations between special subalgebras of a

* References by number are to the bibliography at the end.