find a suitable mapping of V on the complex plane. This is possible if V can be covered by a finite or denumerable number of neighborhoods homeomorphic to a region in the plane, and if V is orientable. The classification follows the standard procedure, due to Jordan for closed surfaces, and to Kerékjártó for open surfaces.

In Chapter 5, the topological characterization of analytic functions is given. A transformation f of the space X into the space X_0 is called *equivalent* to the transformation f' of X' into X_0' if there are topological transformations h of X into X' and h_0 of X_0 into X_0' such that

$$f'(p) = h_0(f(h^{-1}(p))).$$

Two properties of mappings which are invariant under this equivalence are the following: The image of any open set is an open set, and no closed connected set containing more than one point goes into a single point. Mappings with these properties are called *interior*. (This term, or *inner*, is now commonly used by topologists to refer to mappings satisfying the first property.) The fundamental theorem is that any analytic function, as a mapping of its Riemann surface on the complex plane, is interior, and conversely, any interior mapping of a surface on the complex plane is equivalent to an analytic function for which the given surface can be taken as its Riemann surface. The essential step in the proof is to show that an interior transformation behaves locally like z^n for some n, as in (1).

The last chapter gives some applications of preceding methods and results, especially to properties of transformations of one Riemann surface into another. The formula of Hurwitz, relating the genus and number of boundaries of each surface to the degree of the transformation and the total amount of branching, is given and generalized. Asymptotic and limiting values of an analytic function are discussed.

The book should prove of real interest to anyone wishing to study deeply into the underlying topological properties of Riemann surfaces and analytic functions. On the whole, the exposition is quite clear, though here and there one finds slight errors and omissions of important details.

HASSLER WHITNEY

Opérations Infinitésimales Linéaires. By Vito Volterra and Bohuslav Hostinsky. Paris' Gauthiers-Villars, 1938. 7+238 pp.

The infinitesimal calculus of linear operators was invented by Volterra in 1887, and it is with unusual interest that one opens a volume written fifty years later on this important subject, when one discovers that he is a coauthor.

Consider a linear operator X which is a function $X(t)$ of the time t. If multiplication is taken as the fundamental operation,* then analogy with ordinary functions suggests letting the quotient $X(t+\Delta t)X^{-1}(t)$ measure the "change" in X during the interval from t to $t+\Delta t$, and

$$\lim_{\Delta t\to 0} \frac{1}{\Delta t} [X(t + \Delta t)X^{-1}(t)] \tag{1}$$

measure the "rate of change," or "derivative" (more properly, right-derivative) of $X(t)$. It is natural to regard this derivative as a sort of "infinitesimal linear operator," whence the title of the book.

* If addition is taken as the fundamental operation, one gets the (commutative) infinitesimal calculus of vectors, which was discussed by H. Grassmann in 1862, in his *Ausdehnungslehre*, part 2, chaps. 2–4.