ALGEBRAIC POSTULATES AND A GEOMETRIC INTERPRETATION FOR THE LEWIS CALCULUS OF STRICT IMPLICATION

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1. **Two further postulates for a Boolean ring with a unit element.** If addition, subtraction, and multiplication are properly defined in logic, it may be shown* that the postulates for these operations are identical with those in a ring, in which every element is idempotent, satisfying the postulate $xx=x$. Such a ring is called a Boolean ring. The postulates for a Boolean ring with a unit element are therefore the following:

A. *Addition is always possible, commutative, and associative.*
B. *Multiplication is always possible, associative, and both left- and right-distributive with respect to addition.*
C. *Subtraction is always possible.*
D. $xx=x$.
E. *There exists an element* 1 *such that* $x1=x$ *for every element* x *in the ring.*

Here we shall introduce a new operation, represented by x^∞, which satisfies the following two further postulates:

F_1. *For every element* x *there exists an element* x^∞ *such that* $x^\infty x=x^\infty$.
F_2. *For any two elements* x *and* y *we have* $(xy)^\infty = x^\infty y^\infty$.

The postulates A-F_2, obtained above, may be called the algebraic postulates for the Lewis calculus of strict implication.

2. **A geometric meaning of the symbol** x^∞. A geometric meaning† may be attached to x^∞ as follows: Let x be a point set in the euclidean

* See M. H. Stone. *The theory of representations for Boolean algebras*, Transactions of this Society, vol. 40 (1936), pp. 37–53.

† Another geometric meaning of x^∞ may be obtained by assuming 1^∞ to be any one fixed point or any set of fixed points (finite or infinite in number and continuous or discontinuous in character) and setting $x^\infty = x1^\infty$. If we assume that 1^∞ is a fixed point, we have then the following property:

G. x^∞ *is two-valued, that is,* $x^\infty = 1^\infty$ or 0^∞,

which is independent of the postulates A-F_2. This sub-Boolean algebra with the postulates A-G does not become the ordinary two-valued Boolean algebra, unless we assume further that x is two-valued.