POLYGENIC FUNCTIONS WHOSE ASSOCIATED ELEMENT-TO-POINT TRANSFORMATION CONVERTS UNIONS INTO POINTS*

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1. **Introduction.** A function $w=\phi(x, y)+i\psi(x, y)$ is called a *polygenic function* of the complex variable $z=x+iy$ if the real functions ϕ and ψ are general, that is, are not required to satisfy the Cauchy-Riemann equations. The value of the derivative of a polygenic function at a point z_0 depends in general not only on the point z_0 but also on the direction θ along which z approaches z_0; that is, dw/dz is of the form $F(x, y, \theta)$. Thus the derivative $\gamma=dw/dz$ of a polygenic function may be regarded as determining a correspondence between the lineal elements (x, y, θ) of the z-plane and the points (α, β) of the γ-plane, where $\gamma=\alpha+i\beta$. We call this correspondence the *element-to-point transformation T associated with the polygenic function w.*

In previous papers (Kasner, *A new theory of polygenic functions*, Science, vol. 66 (1927), pp. 581–582; *General theory of polygenic functions*, Proceedings of the National Academy of Sciences, vol. 13 (1928), pp. 75–82; *The second derivative of a polygenic function*, Transactions of this Society, vol. 30 (1928), pp. 805–818) we have shown that the element-to-point transformation T associated with a polygenic function must possess the two following properties:

I. *Elements at a given point in the z-plane correspond to points of a circle I in the γ-plane.*

II. *Corresponding central angles of the circle and angles at the point are in the ratio* $-2:1$.

If an element-to-point transformation T possesses the property I, then we define the function $H+iK$, which as a vector represents the center of the circle I, to be the *center function* of T, and the function $(H+h)+i(K+k)$, which as a vector represents the point (called the *initial point* of the circle) on the circle I which corresponds to the initial direction $\theta=0$ in the z-plane, we define to be the *principal phase function* of T. The circle I together with its initial point we call a *clock*.

We then find (Kasner, *A complete characterization of polygenic functions*, Proceedings of the National Academy of Sciences, vol. 22

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