

A NOTE ON THE ELEMENTARY DIVISOR THEORY IN NON-COMMUTATIVE DOMAINS*

TADASI NAKAYAMA

Two main features of the classical elementary divisor theory are the reduction of a matrix with integral coefficients to a normal form, which is a diagonal form with certain properties, and the uniqueness of such a normal form. The former of these two was extended to non-commutative domains by B. L. van der Waerden† and J. H. M. Wedderburn,‡ and a further contribution in this line was made by N. Jacobson.§ Moreover, O. Teichmüller|| showed recently that the so-called euclidean division process is unnecessary for the purpose and the weaker assumption that the domain is a principal ideal domain is sufficient. As for the second problem, namely the uniqueness problem, as it seems to me, little has been done in the non-commutative case except to show that the directly indecomposable components of the diagonal elements as a whole are, in virtue of the Krull-Remak-Schmidt theorem, unique up to similarity. In the present short note¶ we shall, generalizing a result in a joint note of K. Asano and the author,** see that the diagonal elements of a Jacobson-Teichmüller normal form themselves are determined uniquely up to similarity, although this uniqueness theorem is not so satisfactory and is essentially not so far from the uniqueness of the indecomposable components.

Let I be a (not necessarily commutative) domain of integrity†† in

* Presented to the Society, February 26, 1938. The writer is grateful to the referee whose comments enabled him to make some necessary changes.

† B. L. van der Waerden, *Moderne Algebra*, vol. 2, 1st edition, 1932.

‡ J. H. M. Wedderburn, *Non-commutative domains of integrity*, Journal für die reine und angewandte Mathematik, vol. 167 (1932).

§ N. Jacobson, *Pseudolinear transformations*, Annals of Mathematics, vol. 38 (1937).

|| O. Teichmüller, *Der Elementarteilersatz für nichtkommutative Ringe*, Sitzungsberichte der Preussischen Akademie der Wissenschaften, 1937.

¶ The abstract of the note has appeared in this Bulletin, abstract 44-3-118, but the present note consists of the content of the second half of the original one; the first half was omitted since the author found later that that part had been proved in Teichmüller, loc. cit. The note came out originally from an oral discussion with K. Asano and M. Takahasi.

** K. Asano and T. Nakayama, *Über halbbilineare Transformationen*, Mathematische Annalen, vol. 115 (1937).

†† That is, an associative ring with unit element 1 and without divisors of zero.