ENUMERATIVE PROPERTIES OF PLANE CONNECTED n-LINES

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1. **Introduction.** Consider n distinct lines $a_1, a_2, \cdots, a_n$, no two of which are parallel, in a euclidean plane ϕ. These n lines, together with their $n(n-1)/2$ points of intersection, some or all of which may be coincident, form a configuration which we temporarily denote by K. Any one of the points in K, say the point of intersection of the lines a_i and a_j, may be regarded as a virtually non-present intersection of a_i and a_j. Such a point will be called a point of non-connection and will be denoted by Q_{ij}. The lines a_i and a_j are then said to be disconnected or to have virtually no intersection at Q_{ij}. Let d be the number of points of non-connection in K, where

$$(1) \qquad 0 \leqq d \leqq (n-1)(n-2)/2.$$

The condition expressed by (1) will be explained in §2. Any point of K, not regarded as a point of non-connection, will be called a point of connection and will be denoted by P_{ij} if it is the intersection of the lines a_i and a_j. If the d points of non-connection in K are taken in such a way that each of the n lines has one point of connection with at least one of the remaining lines, the resulting configuration is called a plane connected n-line with d points of non-connection and will henceforth be denoted by γ_d^n or just γ.

The object of this paper is to derive some of the enumerative properties of γ. What these properties are will be explained as we proceed. They will all be expressed in terms of n and d.

2. **The maximum number of points of non-connection.** If $d=0$, then all the $n(n-1)/2$ points in γ_0 are points of connection. We may call γ_0^n an absolutely connected n-line. Suppose $d>0$. Obviously no $n-1$ of the d assumed points of non-connection can be on any one line, say a_1. For, if $n-1$ of them did lie on a_1, then a_1 would be disconnected from the remaining lines, and γ would not be a connected n-line. Let there be $n-2$ such points on a_1. Then a_1 is connected with another line, say a_2, by the point P_{12}. If a_2 is to be connected with a third line a_3, then no more than $n-3$ of the remaining points of non-connection can be on a_2. Similarly, if a_3 is to be connected with a fourth line a_4, then a_3 cannot have on it more than $n-4$ of the remaining $d-(n-2)-(n-3)$ points of non-connection. Continuing this proc-