CONCERNING R. L. MOORE'S AXIOM 5*

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In Axiom 5† of his *Foundations of Point Set Theory*, R. L. Moore postulates the existence of a simple closed curve having certain properties. Professor Moore pointed out to me, while I was still in one of his classes, that in some respects this was undesirable, and that if possible it would be better to state a simple axiom without this feature from which Axiom 5 would follow as a theorem in the presence of the other axioms assumed up to this point in *Foundations*, namely, Axioms 0–4. It is the object of this paper to show that this is possible.

Suppose that S, the set of all points, is a space satisfying Axioms 0–4 of Moore's *Foundations* and the following axiom:

AXIOM 5′. *If A is a point of a region R and B is a point distinct from A, there exists in R a compact continuum separating A from B.*

THEOREM 1. *If P is a point of a connected domain D, then $D-P$ is connected.*

PROOF. Suppose that $D-P$ is not connected. There exist two distinct components H and K of $D-P$, each having P on its boundary. Let A and B denote points of H and K, respectively, and let AB denote an arc from A to B in $S-P$.‡ Let R denote a region lying in D, containing P but containing no point of AB. By Axiom 5′, R contains a continuum T separating A from P. The continuum T must, therefore, contain a point of H and a point of K. Since T is a connected subset of $D-P$, this involves a contradiction.

THEOREM 2. *The space S contains no local cut points.*

THEOREM 3. *If A and B are distinct points, there exists a simple closed curve containing A and B.*§

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† *If A is a point of a region R and B is a point distinct from A, there exists, in R, a simple closed curve separating A from B.* Axioms, theorems, and definitions used, but not explicitly stated in this paper, are those of R. L. Moore's *Foundations of Point Set Theory*, American Mathematical Society Colloquium Publications, vol. 13, New York, 1932. This book will be referred to as *Foundations*.

‡ The existence of AB follows from Theorem 1 of Chapter 2 and Axiom 3 of *Foundations*.

§ For an argument to prove Theorem 3 using Theorem 2, the reader is referred to certain portions of the argument for Theorem A on page 54 of the author's paper, *Concerning certain topologically flat spaces*, Transactions of this Society, vol. 42 (1937), pp. 53–93.