METRIC PROPERTIES OF THE CYLINDER OF KUBOTA

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1. **Introduction.** It is the purpose of this note to derive some properties of the surface normal and the *superosculating lines* on an analytic surface S by means of a parabolic cylinder which Kubota has defined* and has used for studying the properties of the affine normal to a surface and the curves of Darboux and Segre in affine differential geometry. If we consider all of the sections of an analytic surface S made by planes passing through the same tangent t, the locus of the parabolas which osculate these sections at the common point of contact P is a parabolic cylinder which we shall call the *cylinder of Kubota.*

In what follows we shall adhere quite closely to the notation used in Chapter 6 of Lane's *Projective Differential Geometry of Curves and Surfaces*, Chicago, 1932.

2. **Analytic basis.** It is convenient to take the lines of curvature for the parametric curves and to employ a local trihedron at a point of the surface whose edges are the tangents of the lines of curvature and the normal of the surface at the point. This section is designed to introduce these concepts and to collect some formulas which will be used later on in this note.

Let us consider in ordinary metric space a non-developable surface, not a sphere, whose parametric equations in cartesian coordinates are

$$x = x(u, v), \qquad y = y(u, v), \qquad z = z(u, v).$$

Let the lines of curvature be the parametric curves on this surface. Then its first and second fundamental forms, written in the customary notation, are

$$E\,du^2 + G\,dv^2, \qquad D\,du^2 + D''dv^2.$$

The principal radii of normal curvature R_1, R_2 at a point of the surface are defined by the formulas $R_1 = E/D$, $R_2 = G/D''$.

As a local trihedron of reference at a point (x, y, z) of the surface S,

* T. Kubota, *Einige Bemerkungen zur Affinflächentheorie*, The Science Reports of the Tôhoku Imperial University, (1), vol. 19 (1930), p. 163. See also T. Kubota, *Einige Bemerkungen zur Affinflächentheorie*, Japanese Journal of Mathematics, vol. 10 (1933), p. 217.