

joining its end points. This is possible by virtue of Lemma 1. Let  $\mathcal{F}$  be a second neighborhood of  $E_0$  interior to  $\mathcal{F}'$  such that every admissible arc  $C$  in  $\mathcal{F}$  joining the points 1 and 2 and satisfying equations (2) cuts the hyperplanes  $x = t_s$  in points  $(t_s, b_{is})$  whose  $y$ -coordinates  $b_{is}$  determine an extremal  $E_b$  of the family (7) lying in  $\mathcal{F}'$ . By Lemma 1 we have  $I_\lambda(C) \geq I_\lambda(E_b)$ , the multipliers  $\lambda_\alpha$  being those belonging to  $E_b$ . But since the arcs  $C$  and  $E_b$  satisfy equations (2), this implies that

$$I_\lambda(C) - I_\lambda(E) = J(C) - J(E_b) \geq 0,$$

the equality holding only in case  $C \equiv E_b$ . Diminish  $\mathcal{F}$  if necessary so that  $J(E_b) \geq J(E_0)$ , as described in Lemma 2. We then have  $J(C) \geq J(E_b) \geq J(E_0)$ , the equality holding in both cases only in case  $C \equiv E_0$ . This proves the theorem.

THE UNIVERSITY OF CHICAGO

## A NEW SUMMATION METHOD FOR DIVERGENT SERIES\*

W. A. MERSMAN

1. **Introduction.** The method to be given here is a modification of that due to Euler-Knopp.† For the weighted means of the partial sums we use the binomial coefficients, but instead of beginning with the first we begin with the "central" one, that is with the greatest. Thus the initial terms always receive the greatest weight, as in the Cesàro-Hölder method.

In this paper it is shown (1) that this new method includes the first two Cesàro methods, and (2) that it also includes the first Euler-Knopp method; further, (3) the exact range of summability of the geometric series is determined. Finally, an example is given which indicates that this method may be more powerful than all those of Cesàro-Hölder, although this statement has not yet been proved.

2. **Definitions and notation.** Throughout we consider a series  $\sum_{k=0}^{\infty} a_k$  and denote by  $S_n$  the sum of its first  $n+1$  terms. We define  $\sigma_n$  as follows:

$$(1) \quad \sigma_n = \frac{1}{4^n} \sum_{k=0}^n C_{2n+1, n-k} S_k,$$

where  $C_{n,k}$  denotes the ordinary binomial coefficient. If  $\sigma_n$  approaches

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† K. Knopp, *Mathematische Zeitschrift*, vol. 15 (1922), pp. 226-253.