

**A SYMMETRIC REPRESENTATION OF THE TWENTY-SEVEN LINES ON A CUBIC SURFACE BY LINES IN A FINITE GEOMETRY\***

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1. **Introduction.** The group  $G_{51840}$  of the automorphisms of the twenty-seven lines and forty-five tritangent planes on a general cubic surface has an even subgroup  $G_{25920}$  which is simple. This may be represented on the one hand as the linear group  $A(4, 3)$ , and on the other hand as the linear group  $HO(4, 4)$ . † Each of these linear groups suggests a representation of the configuration of the lines on the cubic surface by lines in a finite geometry. Coble ‡ has analysed the invariant configurations of the finite projective geometry  $PG(3, 3)$  § under the group  $A(4, 3)$ . In this paper we shall examine those configurations of the  $PG(3, 4)$  under the group  $HO(4, 4)$  which are isomorphic to the configurations of lines and planes on the general cubic surface.

The notation to be developed in this paper assigns coordinates in a symmetric manner to the twenty-seven lines and forty-five tritangent planes on the cubic surface and affords extremely simple conditions to determine their incidences. For this reason it has some advantages over the commonly used double-six notation devised by Schläfli. ||

2. **The planes.** We assign to the planes of the  $PG(3, 4)$  a set of four homogeneous coordinates  $(u_0, u_1, u_2, u_3)$  which are marks of the field  $F \equiv GF(2^2)$ . The four marks of  $F$ , which we denote by 0, 1,  $\omega$ ,  $\bar{\omega}$ , are roots of the congruence

$$(1) \quad u^4 \equiv u \pmod{2}.$$

\* Presented to the Society, February 26, 1938.

† L. E. Dickson, *Linear Groups*, 1901, p. 309. J. S. Frame, *The simple group of order 25920*, *Duke Mathematical Journal*, vol. 2 (1936), p. 477.

‡ A. B. Coble, *A Configuration in Finite Geometry Isomorphic with That of the Twenty-seven Lines of a Cubic Surface*, *Johns Hopkins University Circulars*, no. 208, 1908, pp. 80–88.

§ R. D. Carmichael discusses finite projective geometries in his *Introduction to the Theory of Groups of Finite Order*, 1937.

|| L. Schläfli, *On the twenty-seven lines upon a surface of third order*, *Quarterly Journal of Mathematics*, vol. 2 (1858), pp. 110–120. A. Henderson, *The Twenty-seven Lines upon the Cubic Surface*, 1911. For other notations, see H. S. M. Coxeter, *Polytopes with regular-prismatic vertex figures*, *Philosophical Transactions of the Royal Society of London*, vol. 229 (1930), pp. 396, 418.