A NOTE ON NORMAL DIVISION ALGEBRAS OF PRIME DEGREE*

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Wedderburn has proved† that all normal division algebras of degree three over a non-modular field $\mathfrak{K}$ are cyclic algebras. It is easily verified that his proof is actually correct for $\mathfrak{K}$ of any characteristic not three, and I gave a modification of his proof‡ showing the result also valid for the remaining characteristic three case. Attempts to generalize Wedderburn's proof to algebras of prime degree $p>3$ have thus far been futile, and it is not yet known whether there are any non-cyclic algebras of prime degree. One notes that in both Wedderburn's proof and my modification one starts by studying a non-cyclic cubic field and thus a subfield of a normal splitting field of degree six with a quadratic (cyclic) subfield. I have generalized this property to the case of arbitrary prime degree and have now provided a new proof of the Weddenburn theorem for algebras of degree three in the characteristic three case. The result is the special case $p=3$, $m=2$ of the following theorem:

THEOREM. *Let $\mathfrak{D}$ be a normal division algebra of degree p over a field $\mathfrak{K}$ of characteristic p, and let m be prime to p. Then if $\mathfrak{D}$ has a normal splitting field $\mathfrak{W}$ of degree pm over $\mathfrak{K}$, with a cyclic subfield $\mathfrak{L}$ of degree m over $\mathfrak{K}$, it follows that the algebra $\mathfrak{D}$ is a cyclic algebra.*

In our proof we shall use the following known theorems§ on normal division algebras $\mathfrak{D}$ of degree n over arbitrary fields $\mathfrak{K}$:

LEMMA 1. *Let $\mathfrak{L}$ have degree prime to n. Then $\mathfrak{D}_{\mathfrak{L}}$ is a division algebra.*

LEMMA 2. *Let $\mathfrak{Z}_0$ have degree n over $\mathfrak{K}$ and split $\mathfrak{D}$. Then $\mathfrak{Z}_0$ is equivalent to a (maximal) subfield of $\mathfrak{D}$.*

LEMMA 3. *Let $\mathfrak{D}$ have a cyclic subfield of degree n. Then $\mathfrak{D}$ is a cyclic algebra.*

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† Transactions of this Society, vol. 22 (1921), pp. 129–135.

‡ Transactions of this Society, vol. 36 (1934), pp. 388–394.

§ Cf. Deuring's *Algebren* for our notation and the proofs of the results of Lemmas 1, 2, 3. Lemma 4 was proved by the author for $\mathfrak{K}$ of characteristic not p, Transactions of this Society, vol. 36 (1934), pp. 885–892, and for $\mathfrak{K}$ of characteristic p, ibid., vol. 39 (1936), pp. 183–188.