A slight modification of the example given shows that if N is chosen arbitrarily, there exists a limited region* having at least N distinct points O whose conjugates D lie at infinity.†

Theorem III becomes false if in the hypothesis the region R is not assumed limited, for the reader may verify that no point O of the region R has its conjugate at infinity if R is the entire plane slit along the positive half of the axis of reals from the point $z=0$ to infinity.

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A NOTE ON LINEAR FUNCTIONALS

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1. **Introduction.** The knowledge of the general form of linear functionals in a given abstract space§ is of value in many problems. In some cases (notably in the theory of moment problems) the applications are not to the given space, but to its conjugate space; for example, since the general linear functional on L, the space of functions $x=x(t)$ integrable on (0, 1), has the form

$$(1.1)\quad f(x) = \int_0^1 x(t)a(t)dt,\quad a \text{ measurable and essentially bounded,}\|$$

one can solve the moment problem

$$(1.2)\qquad \mu_n = \int_0^1 t^n a(t)dt, \qquad n = 0, 1, 2, \cdots,$$

for essentially bounded functions a.¶ From the point of view of the theory of moment problems, it seems quite fortuitous that there

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* For the *unlimited* region R: $|y| \leqq b > 0$ of the (x, y)-plane, every point $(x, 0)$ has as conjugate the point at infinity.

† The referee points out that for any region the set of points O whose conjugates D lie at infinity is identical with the set of critical points of the function $r(a)$.

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§ We use the terminology of S. Banach, *Théorie des Opérations Linéaires*, Warsaw, 1932.

‖ S. Banach, op. cit., p. 65. The function a is said to be essentially bounded if there is a number M such that $|a(t)| \leqq M$ for almost all t on (0, 1); we denote by $\sup^0 |a(t)|$ the greatest lower bound of such numbers M.

¶ S. Banach, op. cit., p. 75.