SHORTER NOTICES

Lehrbuch der Gruppentheorie. Vol. 1. By Hans Zassenhaus. (Hamburger Mathematische Einzelschriften, no. 21.) Leipzig and Berlin, Teubner, 1937. 6+151 pp.

This text, mostly on finite discrete groups, was suggested, the author states, by Artin's Hamburg lectures of 1933–1934. Its spirit is that of van der Waerden's *Moderne Algebra*, but of course this specialized book covers much more ground on groups than does that general treatise on modern algebra. Readers familiar with older books on finite groups, or with papers written in the classical manner, will recognize much of the material; the technique of the proofs is frequently simpler, less fortuitous, and more direct than the old. A consistent attempt is made to exploit the principle of homomorphic mapping to the limit. In addition to the practitioners of this technique cited by the author, particularly E. Noether and her many pupils in abstract algebra since about 1921, Gauss should be remembered as its originator. The abstract notions of congruence relations and equivalence relations are seen to have far wider scope than those for which they were devised in the theory of numbers. Dedekind should also be remembered in this connection.

The five chapters of the book present a wealth of material in compact form. The insistence is upon general theorems; the special properties of substitution groups, for instance, receive only passing mention, and linear groups are not discussed. A considerable amount of quite recent material is included; thus there is some account of P. Hall's work on groups whose order is a power of a prime, and Schreier's refinement of the Jordan-Hölder theorem is proved. Specific citations to the literature are likewise recent, and contain references to Ore's work on structures and group theory and G. Birkhoff's on transitive groups. In the historical references, G. A. Miller's reports on certain aspects of group theory and the account in his *Collected Works* might have been included with advantage. As would be expected, most of the references are to modern German work.

As the exposition itself is condensed, we can give only a brief indication of the contents. After a somewhat crowded first chapter of 27 pages dealing with fundamental notions, including the algebra of complexes, the book gets into its stride in the second chapter, devoted to homomorphism (as applied to groups), and groups with operators. This chapter of 45 pages includes, incidentally, the automorphisms of a group, commutative groups, and normal series, also a brief exposition of the groups occurring in algebra. Ideals are here introduced in connection with the subgroups of a skew ring.

One of the author's aims is the construction in a finite number of non-tentative steps of the several algebraic objects of the theory. Accordingly, Chapter 3, of 23 pages, is concerned with the construction of composite groups. The basis theorem for Abelian groups is quickly derived after a brief discussion of direct products (following Fitting, 1934) and some necessary preliminaries on matrices. The chapter concludes with an account of Schreier's Erweiterungstheorie and applications.

The fourth chapter presents Sylow's theorem, its extensions and consequences; and p-groups (groups of prime-power order). It includes a short discussion of Hamiltonian groups. Among the examples is P. Hall's generalized Sylow theorem for finite solvable groups. The concluding chapter, of 15 pages, presents, among other things, monomial representations, the theorems of Grün (1935), groups with all Sylow sub-