ON THE BOUNDARY CONDITION $\partial u/\partial n + au = 0$ FOR HARMONIC FUNCTIONS*

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1. **Introduction.** In a recent paper (referred to below as I)† the author considered the nature of the "reflection" or analytic continuation of harmonic functions across a plane over which the boundary condition

$$\frac{\partial u}{\partial n} + au = 0, \qquad a = \text{const.}, \tag{1}$$

applies. In the following this investigation is continued. First, the case of a spherical (circular in two dimensions) boundary, over which the boundary condition (1) holds is considered. It is shown that singularities admit of a similar "reflection," as in the case of a plane in I; thus a point singularity at P_0 outside a sphere S over which (1) holds is reflected into a point singularity at P_1 (the spherical inverse of P_0 in S) and into a distribution of singularities along the radius vector from the center 0 to P_1.

Returning to plane boundaries, we apply boundary conditions of the form (1) over *two parallel* planes. A rather complicated "reflection" of singularities results, consisting of point singularities as well as of distributed line singularities. The point singularities are located at the periodic row of points obtained by reflecting the original singularity P_0 first in one plane, then in the other one; reflecting these images in the two planes; and so forth. The line singularities are distributed over the straight line through the above point singularities. The density of the distributed line singularities is an analytic function of the distance along the line bearing them between the point singularities, but changes abruptly from one analytic function to another one in crossing these points.

Some of the above features are believed to be typical of analytic continuations of a great variety of expansions in characteristic functions related to two point problems.

2. **Circular and spherical boundaries.** We shall consider the question of reflections of singularities of harmonic functions across spherical or circular boundaries corresponding to the condition (1).

* Presented to the Society, September 5, 1934.

† This Bulletin, vol. 43 (1937), pp. 873–885.