A NOTE ON FREDHOLM-STIELTJES INTEGRAL EQUATIONS*

F. G. DRESSEL

1. **Introduction.** The object of this paper is to show that the integral equation†

$$f(x) = m(x) + \lambda \int_0^1 f(y)dG(x, y), \qquad 0 \leqq x, y \leqq 1, \tag{1}$$

can be changed into an ordinary Fredholm equation when $G(x, y)$ is absolutely continuous $g(y)$.‡ The integration is carried out in the Young-Stieltjes sense, and $g(y)$ is a bounded, monotone increasing function.

2. **Lemmas.** If $h(x)$ is of bounded variation and we set $h(x) = h(0)$, $(x<0)$, and $h(x) = h(1)$, $(x>1)$, then we may define the completely additive function of sets $\bar{h}(e)$ by

$$\bar{h}(e) = h(x_2 + 0) - h(x_1 - 0), \qquad e = e(x_1 \leqq t \leqq x_2).$$

Using this notation we have the following lemma:

LEMMA 1. *If $f(x)$ is measurable Borel then*

$$\int_0^1 f(x)dh(x) = \int_0^1 f(x)d\bar{h},$$

the left side being Young-Stieltjes integration, the right Radon-Stieltjes.

In case one integral does not exist the equality sign is taken to mean that the other integration is non-existent. Because of the properties of the integrals under consideration, we need only prove the equality for the functions

$$\begin{aligned} f_1(x) &= 1, \ x = \alpha, & f_2(x) &= 1, \ 0 \leqq \alpha < x < \beta \leqq 1, \\ &= 0, \ x \neq \alpha; & &= 0, \ \text{elsewhere}. \end{aligned}$$

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† For a discussion of (1) see G. C. Evans and O. Veblen, *The Cambridge Colloquium Lectures on Mathematics*, American Mathematical Society Colloquium Publications, vol. 5, 1922, p. 101.

‡ For terminology see Alfred J. Maria, *Generalized derivatives*, Annals of Mathematics, vol. 28 (1926–1927), pp. 419–432. I am much indebted to Mr. Maria for many valuable suggestions.

All functions used in the present paper are assumed to be measurable Borel.