A THEOREM ON INTERIOR TRANSFORMATIONS*

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In an earlier paper† it has been shown that if $T(A)=B$ is a light interior transformation, that is, a continuous transformation mapping open sets into open sets and mapping no continuum into a single point, and if A is compact, then for every simple arc pq in B and any $p_0 \epsilon T^{-1}(p)$ there exists a simple arc p_0q_0 in A which maps topologically onto pq under T.

In this paper the following extension to arbitrary dendrites (that is, locally connected continua containing no simple closed curves) in B will be made.

THEOREM. *Let $T(A)=B$ be interior and light, where A is compact. For any dendrite D in B and any $x_0 \epsilon T^{-1}(D)$ there exists a dendrite E in A containing x_0 which maps topologically onto D under T.*

PROOF. Since the transformation $TT^{-1}(D)=D$ is interior,‡ clearly there is no loss of generality in assuming that $D=B$. Now let $H=\sum p_iq_i$ be an arc-development of B; that is, for each $i>0$, $p_iq_i\sum_{j=1}^{j=i-1}p_jq_j=q_i$, and p_i, $(i=1, 2, \cdots)$, and all points of $\overline{H}-H$ are end points of B, where we may suppose that $p_0=T(x_0)$. Now by the result quoted above, there exists an arc x_0y_0 such that $T(x_0y_0)=p_0q_0$ and T is topological on x_0y_0. Likewise there exists an arc x_1y_1 in A such that $T(x_1y_1)=p_1q_1$, T is topological on x_1y_1, and furthermore so that $y_1=T^{-1}(q_1)\cdot x_0y_0$. Similarly there is an arc x_2y_2 contained in A so that $T(x_2y_2)=p_2q_2$, T is topological on x_2y_2, and $y_2=T^{-1}(q_2)\cdot\sum_0^1 x_iy_i$. Continuing this process indefinitely we obtain $K=\sum x_iy_i$, so that $T(K)=H$, and T is topological on K. Clearly $\overline{K}$ is a continuum and $T(\overline{K})=B$. Hence our proof will be complete as soon as we show that T is 1 to 1 on $\overline{K}$, or, what amounts to the same thing, that for each $p\epsilon B$, $T^{-1}(p)\cdot\overline{K}$ reduces to a single point.

Now there exists a monotone decreasing sequence of connected neighborhoods $V_1, V_2, V_3, \cdots$ of p in B with $\delta(V_i)\rightarrow 0$. Furthermore,

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† See my paper in the Duke Mathematical Journal, vol. 3 (1937), p. 377, Theorem 4.1. Compare with Stoilow, Annales Scientifiques de l'École Normale Supérieure, vol. 63 (1928), pp. 347–382; and Montgomery, Transactions of this Society, vol. 42 (1937), pp. 328–329.

‡ See my paper, loc. cit., p. 370, Lemma 1.2.