

$$(4)^* \quad p = \left(\frac{a + 2b + 5c + 10d}{6}\right)^2 + 2\left(\frac{a - b + 5c - 5d}{6}\right)^2 \\ + 5\left(\frac{a + 2b - c - 2d}{6}\right)^2 + 10\left(\frac{a - b - c + d}{6}\right)^2.$$

In view of (2) and (3), the numerators in (4) are all even. Then, unless exactly three of  $a$ ,  $b$ ,  $c$ ,  $d$  are divisible by 3, we can choose signs for  $a$ ,  $b$ ,  $c$ ,  $d$  so that

$$(5) \quad a - b - c + d \equiv 0 \pmod{3}.$$

Then all the other numerators in (4) are divisible by 3.

In the exceptional case either  $a$  and  $b$  or  $c$  and  $d$  are divisible by 3. But the identity

$$(6) \quad 9(A^2 + 2B^2) = (A \pm 4B)^2 + 2(2A \mp B)^2$$

(repeated if necessary) shows that any multiple of 3 of the form  $x^2 + 2y^2$  can be expressed in that form with  $x$ ,  $y$  prime to 3. Then (5) can be verified as above, and  $q = 1$ . We have now proved the following theorem:

**THEOREM 4.** *Every positive integer is representable in the form*

$$a^2 + 2b^2 + 5c^2 + 10d^2.$$

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## A MOMENT-GENERATING FUNCTION WHICH IS USEFUL IN SOLVING CERTAIN MATCHING PROBLEMS †

EDWIN G. OLDS

**1. Introduction.** In a book published several years ago, Fry ‡ devoted considerable attention to various aspects of a problem which he called, "the psychic research problem." His introductory problem is the following: "A spiritualistic medium claims to be able to tell the

\* Formula (4) and the rest of the proof of this theorem were suggested by Gordon Pall.

† Presented to the Society and The Institute of Mathematical Statistics, December 30, 1937.

‡ T. C. Fry, *Probability and Its Engineering Uses*, Van Nostrand, New York, 1928, pp. 41-77.