

closed interval with norm the absolute value of the function, and the space of all functions which are Lebesgue integrable to the p th power, $p \geq 1$, with norm the p th root of the integral of the p th power of the absolute value of the function, are all spaces with a denumerable base in the sense of Schauder and Banach, and consequently of type A , the above theorem holds of all completely continuous linear transformations with Banach spaces as domains and such spaces as ranges.*

UNIVERSITY OF MICHIGAN

MULTIVALENT FUNCTIONS OF ORDER $p \dagger$

M. S. ROBERTSON \ddagger

1. **Introduction.** For the class of k -wise symmetric functions ·

$$(1.1) \quad f(z) = \sum_{n=1}^{\infty} a_n z^n, \quad a_1 = 1, \quad a_n = 0 \text{ for } n \not\equiv 1 \pmod{k},$$

which are regular and univalent within the unit circle, it has been conjectured that there exists a constant $A(k)$ so that for all n

$$(1.2) \quad |a_n| \leq A(k)n^{2/k-1}.$$

Proofs of this inequality for $k=1, 2, 2, 3$, were given by J. E. Littlewood, § R. E. A. C. Paley and J. E. Littlewood, ¶ E. Landau, ¶¶ and V. Levin** respectively. As far as the author is aware there is no valid proof †† for $k > 3$ in the literature as yet.

It is the purpose of this note to point out that the methods of proof

* Hildebrandt, this Bulletin, vol. 36 (1931), p. 197.

† Presented to the Society, February 20, 1937.

‡ The author is indebted to the referee for helpful suggestions which led to a revision of this note.

§ See J. E. Littlewood, *On inequalities in the theory of functions*, Proceedings of the London Mathematical Society, (2), vol. 23 (1925), pp. 481–519.

¶ See R. E. A. C. Paley and J. E. Littlewood, *A proof that an odd schlicht function has bounded coefficients*, Journal of the London Mathematical Society, vol. 7 (1932), pp. 167–169.

¶¶ See E. Landau, *Über ungerade schlichte Funktionen*, Mathematische Zeitschrift, vol. 37 (1933), pp. 33–35.

** See V. Levin, *Ein Beitrag zum Koeffizientenproblem der schlichten Funktionen*, Mathematische Zeitschrift, vol. 38 (1934), pp. 306–311.

†† See K. Joh and S. Takahashi, *Ein Beweis für Szegösche Vermutung über schlichte Potenzreihen*, Proceedings of the Imperial Academy of Japan, vol. 10 (1934) pp. 137–139. The proof therein was found to be defective: see Zentralblatt für Mathematik, vol. 9 (1934), pp. 75–76.