MATRIC CONJUGATES IN A RING $R(A)$*

H. S. THURSTON

1. **Introduction.** The concept of conjugate sets of matrices has undergone several modifications since first defined by Taber. Given a matrix M_0, Taber† defined a set $M_1, M_2, \cdots, M_{n-1}$ to be conjugate to M_0 if (a) the M_i are commutative, (b) they have a common characteristic equation, (c) their elementary symmetric functions are scalars and equal to the elementary symmetric functions of the roots of their characteristic equation.

In Taber's paper, the latent roots of M_0 were assumed to be distinct. Franklin‡ generalized the definition so that this restriction is unnecessary. A set $M_1, M_2, \cdots, M_{n-1}$ is conjugate in the sense of Franklin if (a) the M_i are commutative, (b) their elementary symmetric functions are the elementary symmetric functions of the latent roots of M_0.

Further extension of the concept was made by Sokolnikoff.§ Given a matrix M_0 whose minimum equation is $g(x) = \prod_{i=1}^{\nu}(x-\rho_i)^{\pi_i} = 0$, $(\sum \pi_i = m)$, a set $M_1, M_2, \cdots, M_{n-1}$ is conjugate to M_0 with respect to $g(x)=0$, if (a) each M_i is expressible as a polynomial in M_0 with coefficients in the field formed by adjoining the roots ρ_i and the π_ith roots of unity to the field of the elements of M_0, (b) the elementary symmetric functions of the M_i are the elementary symmetric functions of the roots of $g(x)=0$.

Hermann‖ has used the term conjugate in an even broader sense to denote a set of matrices M_i whose elementary symmetric functions are scalars. That is, the M_i are conjugate with respect to any given polynomial $F(x)$ in that their elementary symmetric functions are the elementary symmetric functions of the roots of $F(x)=0$.

In this paper we propose, by use of the principal idempotent elements of a matrix A, to obtain conjugates of each of the above types corresponding to a restricted class of matrices, namely, any given matrix in the ring $R(A)$, where A is any matrix with simple latent roots. The symbolism employed enables us to write down immedi-

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† H. Taber, American Journal of Mathematics, vol. 13 (1891), pp. 157–172.

‡ P. Franklin, Annals of Mathematics, (2), vol. 23 (1921), pp. 97–100.

§ E. S. Sokolnikoff, American Journal of Mathematics, vol. 35 (1933), pp. 167–180.

‖ A. Hermann, Compositio Mathematica, vol. 1 (1934), pp. 284–302.