$$(15)\quad f + \lambda g = \phi(\lambda),\ g = 1 + \sum_{i=2}^{p} \frac{b_i^2}{(\lambda - \lambda_i)^2} - \sum_{j=p+1}^{n} \frac{b_j^2}{(\lambda_j - \lambda)^2} = \phi'(\lambda),$$

a rational function of λ continuous in the interval (11).

If all the $b_k \neq 0$ we have $\phi'(\lambda_{p+1}) = -\infty$, $\phi'(\lambda_p) = \infty$ if $p > 1$, while if $p = 1$, then $\phi'(-\infty) = 1 > 0$. Hence there exists a λ in the intervals (10) such that $\phi'(\lambda) = g = 0$. But then our hypothesis states that $f = \phi(\lambda) > 0$. By (12), and since $\phi(\lambda) > 0$, we have $f + \lambda g$ positive definite.

There remains the case where some $b_k = 0$. Here we may permute the x_i and change the sign of g if necessary and carry the corresponding x_k into x_1. Then $f = -\lambda_1 x_1^2 + f_0(x_2, \cdots, x_n)$. As in the proof above we may carry f_0 into (7) and have f in the form (3). But $f > 0$ for $g = 0$ and as in the proof of Lemma 2 we have (5), and $f + \lambda g$ is positive definite for λ as in (6).

We have proved our theorem. Notice that our reduction to the case g non-singular together with Lemmas 1, 2 determines the range of λ for which $f + \lambda g$ is positive definite.

THE UNIVERSITY OF CHICAGO

THE RIEMANNIAN CURVATURE OF A HYPERSURFACE*

AARON FIALKOW†

1. **Introduction.** It is a well known theorem of Gauss that the total curvature of any two dimensional surface in euclidean three space is equal to the product of the principal normal curvatures. Eisenhart‡ has shown that a generalization of this theorem applies to Riemann spaces of class one; that is, the hypersurfaces of an n-dimensional flat space. He proves the theorem:

When the lines of curvature of a Riemann space V_n of class one are real and none of them is tangent to a null vector, the Riemannian curvature at a point for the orientation determined by the direction of two lines of curvature at the point is numerically equal to the product of the corresponding normal curvatures; the sign is determined by the character of the normal to V_n in the enveloping flat V_{n+1}.

---

* Presented to the Society, September 10, 1937.

† National Research Fellow.

‡ L. P. Eisenhart, *Riemannian Geometry*, 1926, p. 199.