SINGULAR POINT PROBLEMS IN THE THEORY OF LINEAR DIFFERENTIAL EQUATIONS†

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1. **Introduction.** The discussion of the subject indicated in the title is not intended in any way to be encyclopaedic. The object of this address relates to the general problems of determining the character of solutions of equations

$$\text{(A)} \quad L_n(y) \equiv a_0(x)y^{(n)} + a_1(x)y^{(n-1)} + \cdots + a_{n-1}(x)y^{(1)} + a_n(x)y = 0,$$

$$\text{(B)} \quad L_n(x, \lambda; y) \equiv a_0(x, \lambda)y^{(n)} + a_1(x, \lambda)y^{(n-1)} + \cdots + a_n(x, \lambda)y = 0.$$

The $a_i(x)$ in (A) are assumed to be analytic for $|x| \geqq \rho$, $(x \neq \infty)$, being representable by convergent series of the form

$$(1.1) \qquad \alpha_i(x) = x^{n_i/p}[a_{i,0} + a_{i,1}x^{-1/p} + a_{i,2}x^{-2/p} + \cdots],$$
$$n_i,\ p \text{ integers};\ p > 0;\ i = 0, \cdots, n;\ |x| \geqq \rho,$$

or they are supposed to be merely asymptotic to such possibly divergent series, for x in a suitable region extending to infinity. The $a_i(x, \lambda)$ in (B) are assumed to be indefinitely differentiable in x, for x on a closed real interval (a, b), and analytic in λ for $|\lambda| \geqq \rho > 0$, $(\lambda \neq \infty$; λ is a parameter), being representable by convergent series of the form

$$(1.2) \qquad \alpha_i(x, \lambda) = \lambda^{n_i}\sum_{\nu=0}^{\infty} \alpha_{i,\nu}(x)\lambda^{-\nu}, \qquad n_i \text{ integers};\ i = 0, \cdots, n;$$
$$\alpha_{i,\nu}(x) \text{ indefinitely differentiable on } (a, b);\ a \leqq x \leqq b;\ |\lambda| \geqq \rho,$$

or they are supposed to be asymptotic to a finite number of terms, or to infinitely many terms, to such possibly divergent series, when x is on (a, b) and the parameter λ is in a suitable region R extending to infinity.

The investigation for the problems (A) and (B) (relating to equations (A) and (B), respectively) has the purpose of establishing the character of solutions in the complex neighborhood of the singular point of the equation under consideration. In problem (A) the singular point is at $x = \infty$. In problem (B) the singular point considered is at $\lambda = \infty$, (x in (a, b)).

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