CARNAP ON LOGICAL SYNTAX

The Logical Syntax of Language. By Rudolf Carnap. (International Library of Psychology, Philosophy, and Scientific Method.) Translated from the German by Amethe Smeaton (Countess von Zeppelin). New York, Harcourt, Brace, 1937. 16 +352 pp.

The logical syntax of a symbolic language is a study of the formal properties of sentences of that language. It includes the *formation rules* which determine how the symbols of the language can be combined to form sentences, the *transformation rules* which specify when one sentence of the language can be deduced from other sentences, and the other properties of and relations between sentences which can be defined on the basis of these rules. Syntax is a combinatory analysis of expressions, that is, of finite ordered series of symbols. Hence syntax never refers to the meaning of these symbols. Hilbert showed that a clear, formal presentation of the foundations of mathematics must use a metamathematics which is really a syntax of mathematics. The notions of syntax are of central importance for the current growth of mathematical logic.

The present book systematically develops the concepts of syntax, first for two specific Languages I and II, then for an arbitrary language. The specific Language I is a definite ("constructivist" or "finitist") language. It contains the usual calculus of propositions (not, and, implies, $\cdots$) and a Peano arithmetic, with a symbol for 0 and for successor, and with the customary axioms. Variables representing numbers are included, but the quantifiers like "there exists an x" occur only in a limited form, such as "$(\exists x)3(P(x))$," meaning "there exists an x with $x\leqq 3$ such that $P(x)$," and "$(Kx)5(Q(x))$," denoting the smallest $x\leqq 5$ with the property Q.

Language II is a much richer language, and contains everything usually included in a symbolic logic: all of Language I, plus variables for sentences (that is, propositions), variables for predicates, and variables for functors. Such "functors" are functions with any number of arguments of any type. Quantifiers "there exists an x" and "for all x" are used with all these variables. The predicates, which serve also as classes, are classified by the usual (unbranched) type theory, so that a class of numbers is of lower type than a class of classes of numbers. The language so obtained is of interest because it strives for a maximum of flexibility and not, as is often the case, for a minimum of primitive ideas.

Such symbolic languages are ordinarily restricted to symbols defined by means of the primitive symbols of logic and mathematics. Here, in order to make clearer the nature of language and to prepare for a subsequent discussion of the language of science, Carnap allows Languages I and II to contain not only predicates defined in logical terms, but also *descriptive* predicates and functors. One such descriptive symbol is the temperature functor "*te*," which is to be used so that "$te(3)=5$" means "the temperature at the position 3 is 5." Carnap contends that all sentences of physics can be similarly rendered by a "coordinate" language in which the basic symbols are numbers and not names. The general contention seems to neglect the necessity of specifying by name the coordinate system and the scale of measurement to be used.

The syntax of Languages I and II includes the definitions of such important terms as "directly derivable," "demonstrable," and "refutable." In Language I, the specifications under which one sentence is directly derivable from other sentences include the usual rule, that "A_2" and "A_2 implies A_3" give "A_3", in the following form: If the