# METRIC SPACES WITH GEODESIC RICCI CURVES. I

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1. **Introduction.** The problem of determining all Riemannian spaces of three dimensions admitting geodesic Ricci curves has been solved by G. Ricci* and P. Walberer† using, however, different methods. Although they obtained all such $V_3$, the complete explicit determination of all such $V_n$ for $n>3$ does not seem possible because of the increased number and complexity of the differential equations which arise.

In this paper the following two problems related to the above problem will be considered.

In the first problem we suppose given a set of linearly independent vectors‡ $\lambda^i_{a|}$ and wish to determine necessary and sufficient conditions on the $\lambda^i_{a|}$ in order that a set of scalars $\theta_a(\neq 0)$ exist which will define a metric space $V_n$ with a metric determined by

$$g^{ij} = \sum_h e_h \bar{\lambda}^i_{h|} \bar{\lambda}^j_{h|}, \tag{1}$$

where

$$\bar{\lambda}^i_{a|} = \theta_a \lambda^i_{a|}, \tag{2}$$

and $e_h$ $(=\pm 1)$ are arbitrary; and such that the congruences of curves defined by the $\lambda^i_{a|}$ will be geodesics in the $V_n$ thus determined. (The vectors $\bar{\lambda}^i_{a|}$ define the same congruences as do the $\lambda^i_{a|}$, and these congruences form an orthogonal ennuple in the $V_n$.)

In the second problem we assume that these conditions on the $\lambda^i_{a|}$ have been determined and that the $n$ congruences defined by a set of $\lambda^i_{a|}$ are geodesics in the $V_n$ determined by

$$g^{ij} = \sum_h e_h \lambda^i_{h|} \lambda^j_{h|};$$

we then find necessary and sufficient conditions that, with respect to the metric (1), the congruences be geodesic Ricci curves.

* G. Ricci, *Sulle varieta a tre dimensioni dotate die terne principali di congruenze geodetiche*, Rendiconti della Reale Accademia dei Lincei, (5), vol. 27 (1918), pp. 21–28, 75–87.

† P. Walberer, *Riemannsche Raume mit geodätischen Riccikurven*, Hamburger Abhandlungen, vol. 10 (1934), pp. 152–168.

‡ All indices take the values 1, 2, $\cdots$, $n$ unless otherwise noted.