ON THE OPERATIONAL DETERMINATION OF TWO DIMENSIONAL GREEN'S FUNCTIONS IN THE THEORY OF HEAT CONDUCTION†

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1. **Introduction.** In a previous paper‡ the writer has described an operational method for evaluating Green's functions in the theory of heat conduction and illustrated the method for the case of a semi-infinite solid. In this case the starting point was the solution of the differential equation of heat conduction satisfying the condition of a plane source.

It is the object of this paper to illustrate the same method for the case of the two dimensional flow of heat, in which the starting point is the solution of the differential equation of heat conduction satisfying the condition for a line source.

Specifically, we shall determine the Green's functions for the cases where the solid is one of the two following:

(A) An infinite cylinder.

(B) A solid bounded internally by a cylinder.

In both cases we shall take the boundary condition in the form

$$\frac{\partial u}{\partial r} + hu = 0 \qquad \text{for} \qquad r = a.$$

From the general solution to be derived it will be easy, by making $h=0$ or $h=\infty$ in the general solution,§ to obtain the corresponding solutions for the two important cases where the boundary is (1) impervious to heat, (2) kept at 0°.

2. **Case** (A). We start with the solution

$$(1) \qquad u(r, \theta, t; r_0, \theta_0) = \frac{1}{4\pi kt} \exp\left\{-\frac{r^2 + r_0^2 - 2rr_0 \cos(\theta - \theta_0)}{4kt}\right\}$$

which satisfies the condition for a line source at (r_0, θ_0). The solution (1) may be written in the equivalent forms

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‡ Philosophical Magazine, (7), vol. 24 (1937), pp. 62–70.

§ Some special cases of the problems discussed in this paper have been treated by S. Goldstein, Proceedings of the London Mathematical Society, (2), vol. 34 (1932), pp. 51–88. Goldstein treats the case where the line source coincides with the axis of the cylinder. His boundary condition is $u=0$ or $\partial u/\partial r=0$.