DIFFERENTIAL INVARIANT THEORY OF ALTERNATING TENSORS*

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1. **Introduction.** In a former paper† a general method was developed for obtaining a complete system of tensors for a general n-ary q-ic differential form. The quantities $\Lambda_{r_1 \cdots r_q s}$ of that article are proportional to the quantities $a_{r_1 \cdots r_q s}$ of this paper which do not contain the second derivatives when the fundamental tensor is alternating. Thus that method for establishing convariant differentiation with respect to a covariant q-ic form fails when the form is alternating. This exceptional case will be treated here.

Under an analytic transformation of coordinates

$$x^i = x^i(\bar{x}), \qquad i = 1, \cdots, n, \qquad \left| \frac{\partial x^r}{\partial \bar{x}^s} \right| \neq 0, \tag{1}$$

the alternating covariant tensor $a_{r_1 \cdots r_q}$ transforms by the equations

$$\bar{a}_{r_1 \cdots r_q} = a_{p_1 \cdots p_q} p_{r_1}^{p_1} \cdots p_{r_q}^{p_q}, \qquad p_s^r = \frac{\partial x^r}{\partial \bar{x}^s}. \tag{2}$$

The property of being alternating is invariant.

We propose to find conditions under which these equations with preassigned $\bar{a}_{r_1 \cdots r_q}$ and $a_{r_1 \cdots r_q}$ admit solutions $p_s{}^r$, $|p_s{}^r| \neq 0$,

$$p_{st}^r = p_{ts}^r, \qquad p_{st}^r \equiv \frac{\partial p_s^r}{\partial \bar{x}^t}, \tag{3}$$

and for which the differential equations

$$\frac{\partial x^r}{\partial \bar{x}^s} = p_s^r \tag{4}$$

are integrable and yield solutions (1) determining a transformation of coordinates.

The statement of the conditions under which such systems admit solutions is contained in a note by the writer which precedes the

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† *The invariants of an n-ary q-ic differential form*, Annals of Mathematics, (2), vol. 31 (1930), pp. 134–150.