ON CONTINUED FRACTIONS REPRESENTING CONSTANTS*

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1. **Introduction.** Let $\xi: x^{(1)}, x^{(2)}, x^{(3)}, \cdots$ be an infinite sequence of points $x=(x_1, x_2, x_3, \cdots, x_m)$ in a space S, and let $\phi_1(x), \phi_2(x), \phi_3(x), \cdots, \phi_k(x)$ be single-valued real or complex functions over S. Then the functionally periodic continued fraction

$$1+\frac{\phi_1(x^{(1)})}{1}+\frac{\phi_2(x^{(1)})}{1}+\cdots+\frac{\phi_k(x^{(1)})}{1}+\frac{\phi_1(x^{(2)})}{1}+\cdots$$
$$+\frac{\phi_k(x^{(2)})}{1}+\frac{\phi_1(x^{(3)})}{1}+\cdots$$

is a function $f(\xi)$ of the sequence ξ. By a neighborhood of a sequence ξ: $x^{(1)}, x^{(2)}, x^{(3)}, \cdots$, we shall understand a set N_ξ of sequences subject to the following conditions: (i) ξ is in N_ξ; (ii) if $\eta: y^{(1)}, y^{(2)}, y^{(3)}, \cdots$ is in N_ξ, then $\eta_\nu: y^{(\nu+1)}, y^{(\nu+2)}, y^{(\nu+3)}, \cdots$ and $\zeta_\nu: y^{(1)}, y^{(2)}, y^{(3)}, \cdots, y^{(\nu)}, x^{(\nu+1)}, x^{(\nu+2)}, x^{(\nu+3)}, \cdots$ are in N_ξ for $\nu=1, 2, 3, \cdots$.

Let $A_n(\xi)$ and $B_n(\xi)$ be the numerator and denominator, respectively, of the nth convergent of $f(\xi)$ as computed by means of the usual recursion formulas. Put

$$L(\xi, t) = B_{k-1}(\xi)t^2 + [\phi_k(x^{(1)})B_{k-2}(\xi) - A_{k-1}(\xi)]t - \phi_k(x^{(1)})A_{k-2}(\xi).$$

Then our principal theorem is as follows:

THEOREM 1. *Let there be a sequence* c: $c^{(1)}, c^{(2)}, c^{(3)}, \cdots$, *and a neighborhood* N_c *of* c, *and a number* r *having the following properties*:

(a) $f(\xi)$ *converges uniformly over* N_c,

(b) $f(c)=r$,

(c) $L(\xi, r)=0$ *for every sequence* ξ *in* N_c,

(d) $\phi_i(x^{(\nu)})\neq 0$, $(\nu=1, 2, 3, \cdots; i=1, 2, 3, \cdots, k)$, *for every sequence* ξ: $x^{(1)}, x^{(2)}, x^{(3)}, \cdots$ *in* N_c.

When these conditions are fulfilled, $f(\xi)=r$ *throughout* N_c.

The proof of Theorem 1 is contained in §2; §3 contains a specialization and §4 an application of this theorem. In §5 continued fractions

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