CARTAN ON PROJECTIVELY CONNECTED SPACES

Leçons sur la Théorie des Espaces à Connexion Projective. By Élie Cartan. Edited by P. Vincensini. (Cahiers Scientifiques, publiés sous la direction de M. Gaston Julia, no. 17.) Paris, Gauthier-Villars, 1937. vi+308 pp.

In a recent monograph, La Méthode de Repère mobile, · · · (Actualités Scientifiques et Industrielles, no. 194; reviewed in this Bulletin, November, 1935) E. Cartan presented a splendid outline of his general method of approach toward all branches of differential geometry. In a word, it consists of a far-reaching generalization of the familiar moving trihedral with assistance from the theory of groups. The analytical formulation employs the exterior differential calculus, a discipline extensively used by Cartan since the turn of the century. And there is frequent recourse to the theory of Pfaffian systems. Undoubtedly his unusual analytical machinery is, to many, a source of difficulty. Most differential geometers use, instead, Ricci's tensor calculus and theorems on total differential equations stemming from Christoffel. But every disciple of Ricci knows the profit which lies in the study of Cartan.

In the above monograph the author devoted one paragraph to showing how projective differential geometry fits into his general scheme. In the work here under review this paragraph is presented to us in more satisfactory form as a book of three hundred odd pages.

The book has two principal divisions, the first devoted to classical projective differential geometry, the second, to the geometry of projectively connected spaces. The first chapter is concerned with the projective line, both real and complex. Since from the intrinsic standpoint any two one-dimensional loci are locally equivalent, the purely differential-geometric discussion of the real line is trivial. But by introducing motion, Cartan finds a kinematical theory which is a good introduction to his method of moving reference systems, systems which in this book are always simplexes defining homogeneous projective coordinates. A pleasing detail is his interpretation of the Schwarzian derivative as a projective acceleration. The material on the complex line shows how moving reference systems can be used in the complex domain. This subject he has treated at length in a previous work.

In the second chapter the elements of the theory of plane curves is first developed by Wilczynski's method. Here the author adds an exciting definition of the projective arc. Then, returning to his own method of moving reference systems, he redevelops the theory twice and in some detail, first with aid from the method of reduced equations, and finally directly. In the direct method geometrical intuition is ignored and the invariant theory is developed purely analytically. This attack makes for difficulty but has the advantage of being applicable in general situations where intuition fails to suggest shortcuts. In the present case it leads to an intrinsic reference simplex of the sixth order associated with each point of a curve, in terms of which the projective analogues of the Frenet equations take their simplest form. The chapter ends with a derivation of the so-called structure equations of the projective group and a simplification of the direct method by their use. At one point in this chapter the author makes the surprising statement that curves of constant projective curvature (for example, $y=x^3$) have no inflection points.

The third chapter carries the first division of the work to its conclusion with a discussion of surfaces in three-space. The method of reduced equations leads to the