ON REFLECTION OF SINGULARITIES OF HARMONIC FUNCTIONS CORRESPONDING TO THE BOUNDARY CONDITION $\partial u/\partial n+au=0$

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1. *Introduction.* Familiar "reflection" principles across a plane at which a harmonic function u satisfies either of the two boundary conditions

$$u = 0, \tag{1}$$

$$\partial u/\partial n = 0, \tag{2}$$

where $\partial/\partial n$ denotes the normal derivative, extend the function u from one side of the plane to the other one by means of its negative or positive image respectively. In particular, the singularities of u to one side of the plane are also reflected into their negative or positive images.

In the following we consider the nature of the "reflection" or analytic continuation of a harmonic function u across a plane boundary corresponding to what is perhaps the next simplest boundary condition, namely:

$$\frac{\partial u}{\partial n} + au = 0, \tag{3}$$

where a is a constant. It is shown that the image of each singularity S_0 of u is relatively complex and consists of

(a) a positive image S_1 of S_0 in the boundary plane;

(b) an exponential trail of negative images along the line through S_0 and S_1, beyond S_1, and totalling in amount double the negative of S_0.

Results similar to the above are established for other differential equations; for instance, for the equation of heat conduction. Conditions with higher order derivatives are also considered.

Aside from the interest of the subject matter in connection with analytic continuation, as well as from the point of view of general curiosity that makes one "peep behind the looking glass," the subject is also of interest in view of several pos-