

## ON CERTAIN MATRICES AND THEIR DETERMINANTS\*

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1. *Introduction.* It is well known that certain determinants,  $|c_{ij}|$ , ( $i, j=1, 2, \dots, r$ ), are expressible as the products of  $r$  linear factors of the form  $a_{i1}\omega_{i1} + a_{i2}\omega_{i2} + \dots + a_{ir}\omega_{ir}$ , ( $i=1, 2, \dots, r$ ), where  $a_{ij}$  are rational functions of the elements,  $c_{ij}$ , in the first row of  $|c_{ij}|$ , and where  $\omega_{ij}$  are functions of certain roots of unity and depend only on the relations existing between the elements of succeeding rows and those of the first row of the given determinants. Cyclic determinants and some related types which have this property are well known. †

In the present paper a generalization of these results is carried forward in two directions; first, we replace the scalar elements in the array of the given determinant by square matrices of order  $n$ ; and second, we permit succeeding rows of the array to be other than permutations of the matrices occurring say in the first row of the given determinant. Certain special types, which we here generalize, have been studied by Puchta, Noether, Baltzer, Drude, Burnside, Scorza, and others. ‡

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† Pascal, *Die Determinanten*, §§20–21. Gegenbauer, *Über eine specielle symmetrische Determinante*, Sitzungsberichte, Akademie der Wissenschaften, Mathematische-Naturwissenschaftliche Klasse, vol. 82, II–III (1880), pp. 938–942. Burnside, *On a property of certain determinants*, Messenger of Mathematics, vol. 23 (1894), pp. 112–114.

‡ Puchta, *Ein Determinantensatz und seine Umkehrung*, Denkschriften der Wiener Akademie, vol. 38, 2<sup>te</sup> Abth. (1878), pp. 215–221; *Ein neuer Satz aus der Theorie der Determinanten*, *ibid.*, vol. 44, 2<sup>te</sup> Abth. (1882), pp. 227–282.

Noether, *Zur Theorie der Thetafunctionen*, Mathematische Annalen, vol. 16 (1880), pp. 322–325; *Notiz über eine Classe symmetrischer Determinanten*, *ibid.*, pp. 551–555.

Baltzer, *Ueber einen Satz aus der Determinantentheorie*, Nachrichten, Königlich Gesellschaft der Wissenschaften, Göttingen, 1887, pp. 389–391.

Drude, *Ein Satz aus der Determinantentheorie*, *ibid.*, pp. 118–122.

Burnside, *loc. cit.*

Scorza, *Sopra una certa classe di determinanti e sulla forma Hermitiane*, Giornale di Matematica, vol. 51 (1913), pp. 335–342.