$$\{[f(x+(D-1)\theta)-f(x-\theta D)]F(y)\}_{y=0}=\theta\frac{df(x)}{dx}\cdot$$

Blissard's remark, "*An equation which has a representative quantity is not susceptible to any algebraic operation by which the indices would be affected,*" becomes

$$(Df)^2\neq D^2f.$$

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ON FOURTH ORDER SELF-ADJOINT DIFFERENCE SYSTEMS*

BY V. V. LATSHAW

A linear difference expression for which the differential transform is self-adjoint (anti-self-adjoint) we shall call self-adjoint (anti-self-adjoint).† We choose two fourth order difference equations

$$(1)\quad \begin{aligned}L^+(u)&\equiv p(x)[u(x+2)+u(x-2)]\\&\quad+\lambda[u(x+1)+u(x-1)]+R(x)u(x)=0,\end{aligned}$$

$$(2)\quad \begin{aligned}L^-(u)&\equiv p(x)[u(x+2)-u(x-2)]\\&\quad+\lambda[u(x+1)-u(x-1)]=0,\end{aligned}$$

where $L^+(u)$ is self-adjoint and $L^-(u)$ anti-self-adjoint for the range $(x=a, a+1, \cdots, b-1; b-a\geqq 4)$. $R(x)$ and $p(x)$ are both real, $p(x)$ being a non-vanishing periodic function of period two; λ is a parameter.

Let the functions (y_1, y_2, y_3, y_4) constitute a fundamental set of solutions for either (1) or (2), and (w_1, w_2, w_3, w_4) the set adjoint to it. The two sets are related by the equations

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* Presented to the Society, October 30, 1937.

† J. Kaucky, *Sur les équations aux différences finies qui sont identiques à leurs adjointes*, Publications of the Faculty of Sciences, University of Masaryk, No. 22 (1922). For a discussion of adjoint differential expressions of infinite order, see H. T. Davis, *The Theory of Linear Operators*, 1936, pp. 474–475.