ABSTRACTS OF PAPERS

SUBMITTED FOR PRESENTATION TO THE SOCIETY

The following papers have been submitted to the Secretary and the Associate Secretaries of the Society for presentation at meetings of the Society. They are numbered serially throughout this volume. Cross-references to them in the reports of the meetings will give the number of this volume, the number of this issue, and the serial number of the abstract.

372. Professor R. P. Agnew: *Linear functionals satisfying prescribed conditions.*

Let P be the class of p-functions $p=p(x)$ and F the class of linear functionals $f=f(x)$, defined over a linear space E. A functional $p\varepsilon P$ is said to *enforce* a set S of properties if each $f\varepsilon F$ with $f(x)\leqq p(x)$ for all $x\varepsilon E$ has the properties S. It is shown that $p\varepsilon P$ exists which enforces S if and only if $f\varepsilon F$ exists having properties S, and that $p_2\varepsilon P$ enforces all the properties which $p_1\varepsilon P$ enforces if and only if $p_2(x)\leqq p_1(x)$ for all $x\varepsilon E$. The main problem of this paper is that of characterizing analytically the class of $p\varepsilon P$ which enforces a preassigned property or set of properties. The problem is solved for the property 1^0: $f(x)\leqq p_0(x)$, $p_0\varepsilon P$ being preassigned; for the property 2^0: $f(y)=f(x)$ for all pairs $\{x, y\}$ belonging to a preassigned set Ψ of pairs; and for the property 3^0: both 1^0 and 2^0. The analytic criteria are discussed for the case where Ψ is the set of all pairs $\{x, g(x)\}$ with $x\varepsilon E$, and $g\varepsilon G$ where G is a group of linear transformations mapping E into itself, and for the further special case where G is solvable. Applications to generalized limits and integrals are given. (Received September 25, 1937.)

373. Dr. R. P. Boas and Professor D. V. Widder: *The iterated Stieltjes transform.*

The iterated Stieltjes transform is (1) $f(x)=\int_{0+}(x+u)^{-1}du\int_{0+}^{\infty}(u+t)^{-1}d\alpha(t)$. It is formally equivalent to the S_2-transform (2) $f(x)=\int_{0+}^{\infty}\log(x/t)(x-t)^{-1}d\alpha(t)$; this formal equivalence is not always valid, but (2) can always be written in the form (1). The transform (1) is inverted by the use of a differential operator $H_{k,t}[f(x)]=L_{k,t}L_{k,t}[f(x)]$, where $L_{k,t}[f(x)]$ is the inversion operator for the Stieltjes transform, introduced by D. V. Widder (abstract 43-1-87). If $\alpha(t)$ is the integral of a function $\phi(t)$, $H_{k,t}[f(x)]$ approaches $\phi(t)$ for almost all positive t, $(k\rightarrow\infty)$. If $\alpha(t)$ is a normalized function, of bounded variation on every interval (ϵ, R), $(0<\epsilon<R<\infty)$, the integral of $H_{k,t}[f(x)]$ on $(0, t)$ approaches $\alpha(t)-\alpha(0+)$ for every positive t. Necessary and sufficient conditions for a given $f(x)$ to have the representations (1) and (2) with $\alpha(t)$ belonging to various functional classes are obtained in terms of $H_{k,t}[f(x)]$. For a preliminary report on the S_2-transform, see D. V. Widder, Proceedings of the National Academy of Sciences, vol. 23 (1937), pp. 242–244. (Received September 18, 1937.)