PROOF OF THE NON-ISOMORPHISM OF TWO COLLINEATION GROUPS OF ORDER 5184*

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Introduction. Let S denote the collineation

$$\rho x_r = \epsilon^{r-1} x_r', \quad (r = 1, \cdots, n), \qquad \epsilon = \cos(2\pi/n) + i \sin(2\pi/n),$$

and T the collineation

$$\rho x_r = x_{r+1}', \qquad (r = 1, \cdots, n), \qquad x_{n+1}' \equiv x_1'.$$

The abelian group $\{S, T\}$ of order n^2 is invariant under a group† C_n of order

$$n^5\left(1 - \frac{1}{p_1^2}\right)\left(1 - \frac{1}{p_2^2}\right) \cdots \left(1 - \frac{1}{p_m^2}\right),$$

where $p_1, p_2, \cdots, p_m$ are the distinct prime factors of n. The order of C_6 is 5184.

Winger‡ has discussed briefly the monomial group of order $(r+1)!n^r$ that leaves invariant the variety

$$x_0^n + x_1^n + x_2^n + \cdots + x_r^n = 0.$$

This group is generated by the symmetric group of degree $r+1$ and an abelian group of order n^r in canonical form. For $r=3$ and $n=6$ there results a group G of order 5184 which has been treated by Musselman.§ The purpose of this note is to prove that G and C_6 are not simply isomorphic. The proof consists in showing that the number of collineations of period 2 in G exceeds the number of collineations of period 2 in C_6.

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† In fact, C_n is the largest collineation group in n variables containing $\{S, T\}$ invariantly, the coefficients and variables being in the field of complex numbers. (Author's dissertation, Ohio State University, 1934.)

‡ *Trinomial curves and monomial groups*, American Journal of Mathematics, vol. 52 (1930), p. 394.

§ *On an imprimitive group of order* 5184, American Journal of Mathematics, vol. 49 (1927).