p-ALGEBRAS OVER A FIELD GENERATED BY ONE INDETERMINATE*

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1. *Introduction.* The structure of all division algebras over the simplest type of non-modular field, the field of all rational numbers, has been determined.† The correspondingly simplest type of infinite modular field‡ is the simple transcendental extension $K=F(x)$ of a finite field F. Every division algebra D over such a K is a normal division algebra of degree n over a centrum G which is algebraic of finite degree over K. It is well known that the problem of determining the structure of D is reducible to the case where n is a power of a prime p. When p is the characteristic of F the algebra D is called a p-algebra and we shall solve the problem in this case. Our results will be valid if we replace the finite field F by any perfect field of characteristic p.

The theorem we shall obtain is remarkable not merely because of the character of the result thus derived but also because of the extremely elementary nature of the proof. By using a simple property of the field G described above we shall show that every p-algebra with centrum G is cyclic and of exponent equal to its degree. Moreover this result is due to the unusual fact that all cyclic algebras over G of the same degree p^e have a common pure inseparable splitting field.

2. *Simple Transcendental Extensions of F.* Consider any perfect field F of characteristic p. Then every a of F has the form $a=b^{p^k}$ for b in F. It is easily seen that in fact the correspondences

$$a \longleftrightarrow a^{p^k}, \qquad (k=0,1,\cdots),$$

are automorphisms of F.

We let x be an indeterminate over F, $J=F[x]$ be the set of

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† Cf. the paper of H. Hasse and the author, Transactions of this Society, vol. 34 (1932), pp. 722–726.

‡ There is no structure problem for division algebras over finite fields as they are always finite fields.