ON SYMMETRIC DETERMINANTS

BY W. V. PARKER

In a former paper* the writer proved the following theorem:

THEOREM A. *If* $D=|a_{ij}|$ *is a symmetric determinant of order* $n>4$ *with* a_{ij} *real and* $a_{ii}=0$, $(i=1, 2, \cdots, n)$, *and if all fourth-order principal minors of* D *are zero, then* D *vanishes.*

The purpose of this note is to give some results which are obtained immediately from this theorem and which are in one sense a generalization of this theorem.

Suppose D is a symmetric determinant of order $n>4$, with *real* elements, in which all principal minors of order $n-1$ and also all principal minors of order $n-4$ are zero. If $D'=|A_{ij}|$ is the adjoint of D, then $A_{ii}=0$, $(i=1, 2, \cdots, n)$. Each fourth-order principal minor of D' is equal to the product of D^3 by a principal minor of D of order $n-4$.† Therefore D' satisfies the conditions of Theorem A and hence is zero. But $D'=D^{n-1}$ and hence D is also zero and we have the following theorem:

THEOREM 1. *If* D *is a symmetric determinant of order* $n>4$, *with real elements, in which all principal minors of order* $n-1$ *and also all principal minors of order* $n-4$ *are zero, then* D *vanishes.*

Suppose D is a symmetric determinant of order $n>4$, with *real* elements, in which all principal minors of some order $k>3$ and also all principal minors of order $k-3$ are zero. Let M be any $(k+1)$-rowed principal minor of D, ($M=D$ if $n=5$), then M is a determinant satisfying the conditions of Theorem 1 and hence M is zero. Therefore, in D, all principal minors of order k and also all principal minors of order $k+1$ are zero, hence D is of rank $k-1$ or less.‡ We have thus proved the following theorem:

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* *On real symmetric determinants whose principal diagonal elements are zero*, this Bulletin, vol. 38 (1932), pp. 259–262. See also, *On symmetric determinants*, American Mathematical Monthly, vol. 41 (1934), pp. 174–178.

† Bôcher, *Introduction to Higher Algebra*, p. 31.

‡ Bôcher, loc. cit., page 57, Theorem 2.