THE RESULTANT MATRIX OF TWO POLYNOMIALS*

BY M. M. FLOOD

1. *Introduction.* Frobenius† has shown that if P is a matrix whose characteristic function is $P(x)$ and if $P_0(x)$ is a second polynomial, then their resultant is the determinant of the matrix $P_0(P)$. In particular, if P is non-derogatory,‡ the present author§ has shown that the degree of the highest common factor of $P(x)$ and $P_0(x)$ is the same as the nullity‖ of $P_0(P)$.

In this paper the matrix P is taken to be the companion matrix¶ of $P(x)$, and it is shown that all the remainders in the euclidean algorithm for $P(x)$ and $P_0(x)$ can easily be found from the "resultant matrix" $P_0(P)$. The proof is strictly rational and quite elementary. Finally, the results are applied to a numerical example.

2. *The Algorithm.* The euclidean algorithm for the polynomials $P_0(x)$ and $P_1(x)=P(x)$ may be written in the form

$$(1)\qquad P_{k-1}(x) = R_k(x)P_k(x) - P_{k+1}(x), \qquad (k = 1, 2, \cdots, r),$$

where $P_{r+1}(x)=0$, and the degree of $P_{k+1}(x)$ is less than the degree of $P_k(x)$. Set $S_1(x)=1$, $S_2(x)=R_1(x)$, $P_{-1}(x)=0$, $P_{-2}(x)=1$, and define polynomials $S_k(x)$ and $P_{-k}(x)$ by the relations

$$\left.\begin{aligned} S_{k+1}(x) &= R_k(x)S_k(x) - S_{k-1}(x) \\ P_{-(k+1)}(x) &= R_k(x)P_{-k}(x) - P_{-(k-1)}(x) \end{aligned}\right\}, \quad (k = 2, 3, \cdots, r).$$

A simple induction** now yields the identities

$$(2)\quad P_k(x) = S_k(x)P_1(x) - P_{-k}(x)P_0(x), \quad (k = 1, 2, \cdots, r+1).$$

* Presented to the Society, February 29, 1936. A special case of the principal result of this paper was considered by the present author in a paper having the same title and published in the American Mathematical Monthly, vol. 44 (1937), p. 309.

† Frobenius, Journal für Mathematik, vol. 84 (1878), p. 11.

‡ Sylvester has called a matrix "non-derogatory" when its characteristic function and minimum function are the same.

§ American Mathematical Monthly, vol. 43 (1936), p. 562.

‖ The "nullity" of a matrix is the difference between its order and rank

¶ The "companion matrix" of $P(x)$ is the matrix P_1 defined in §3.

** Netto, *Vorlesungen über Algebra*, §62.