ON A THEOREM OF HIGHER RECIPROCITY*

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1. *Introduction*. Let $\mathfrak{D}$ denote the totality of polynomials in an indeterminate x, with coefficients in a fixed Galois field $GF(p^\pi)$ of order p^π. Let P be a primary irreducible polynomial in $\mathfrak{D}$; then, if A is any polynomial in $\mathfrak{D}$ not divisible by P, we define $\{A|P\}$ as that element in $GF(p^\pi)$ for which

$$\left\{\frac{A}{P}\right\} \equiv A^{(p^{\pi\nu}-1)/(p^\pi-1)} \pmod{P},$$

where ν is the degree of P.

We have then the following theorem of reciprocity due to H. Kuhne‡ and rediscovered by Schmidt§ and Carlitz.‖

If P and Q are primary irreducible polynomials in $\mathfrak{D}$ of degree ν and ρ respectively, then

$$\left\{\frac{P}{Q}\right\} = (-1)^{\rho\nu}\left\{\frac{Q}{P}\right\}.$$

If $M=P_1^{a_1}\cdots P_k^{a_k}$ and $(A, M)=1$ we use the definition,

$$\left\{\frac{A}{M}\right\} = \left\{\frac{A}{P_1}\right\}^{a_1}\cdots\left\{\frac{A}{P_k}\right\}^{a_k}.$$

The purpose of this note is to give a simple new proof of the following theorem:

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‡ H. Kuhne, *Eine Wechselbeziehung zwischen Funktionen mehrerer Unbestimmter die zu Reziprozitätsgesetzen führt*, Journal für die reine und angewandte Mathematik, vol. 124 (1901–02), pp. 121–133.

§ F. K. Schmidt, *Zur Zahlentheorie in Körpern von der Charakteristik p*, Sitzungsberichte der Physikalish-medizinischen Societät zu Erlangen, vol. 58–59 (1928), pp. 159–172.

‖ L. Carlitz, *The arithmetic of polynomials in a Galois field*, American Journal of Mathematics, vol. 54 (1932), pp. 39–50.