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WELLS COLLEGE

JENSEN'S INEQUALITY*

BY E. J. MCSHANE

The simplest form of Jensen's inequality is that if $\phi(x)$ is a convex function, and m is the arithmetic mean of $x_1, \cdots, x_n$, then the mean of the numbers $\phi(x_n)$ is not less than $\phi(m)$. This inequality can be generalized in several different ways. The function $\phi(x)$ can be replaced by a convex function of several variables, and the arithmetic mean can be replaced by any one of several other means, as has been shown in various proofs. Since the inequality is of considerable utility, it seems worth while to have it established in a form which is general enough to cover a wide assortment of applications.

The proofs will rest on two well known properties of convex sets.† If K is closed and convex and a point p is not in K, then p can be separated from K by a hyperplane. If K is closed and convex and p is a boundary point of K, there is a hyperplane of support of K passing through p.

1. *The Inequality in Geometric and in Analytic Form.* It will be convenient in the following proofs to use these symbols and definitions:

R_n is n-dimensional Euclidean space. Its points will be denoted by $(z_1, \cdots, z_n)$ or by z. Linear functions $\sum a_i z_i$ or R_n will be symbolized by $l(z)$.

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† A set is convex if for every pair P, Q of points of the set the line segment PQ is contained in the set.