MODERN METHODS OF ANALYSIS IN POTENTIAL THEORY*

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I shall take the liberty of interpreting the title in terms of a short discussion of certain problems that have been of recent interest, particularly to younger mathematicians, namely, the discontinuous boundary value problems of the Dirichlet and Neumann types, the mixed problem, and finally the generalizations of the potential integral itself to others in which the integrand $1/r$ is replaced by $1/r^\alpha$, $0<\alpha<3$. In this way my review will serve as a continuation of that of the late Professor Kellogg, but will diverge from it in the sort of problem to be considered.†

I. Dirichlet and Neumann Problems

1. *Plane Regions*. It is a natural generalization of Poisson's integral to write it in the form of a Stieltjes integral,

$$(1) \qquad u(r, \theta) = u(M) = \frac{1}{2\pi}\int_0^{2\pi} \frac{a^2 - r^2}{a^2 + r^2 - 2ar\cos(\phi - \theta)}\, dF(\phi),$$

where $F(\phi)$ is periodic and of bounded variation. If the $dF(\phi)$ is replaced by $df(e)$ or $f(de)$ the integral is interpreted as integration with respect to an additive function of sets on the circumference, or in other words, with respect to an arbitrary distribution of positive and negative mass on the circumference, finite in total absolute amount.

The integrand $(a^2-r^2)/[2\pi(a^2+r^2-2ar\cos(\phi-\theta))]$ may also be written in terms of the Green's functions $g(M, P)$ or its conjugate $h(M, P)$, in which P is the point on which is performed the integration. In fact, (1) may be written in the form

$$(2) \qquad u(M) = \int_C \frac{dh(M, P)}{dh(O, P)}\, df(e_P)$$

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† O. D. Kellogg, *Recent progress with the Dirichlet problem*, this Bulletin, vol. 32 (1926), pp. 601–625.