HADAMARD'S MEMORIAL VOLUME

Selecta. Jubilé Scientifique de M. Jacques Hadamard. Paris, Gauthier-Villars, 1935. 432 pp.

The task of selecting a group of representative papers from the writings of a mathematician as prolific and versatile as Hadamard is not an easy one. Obviously, papers containing fundamental results which have attracted almost universal attention should be included. On the other hand, a certain unity, showing a concept at different stages of development, is also desirable if such a volume is to prove useful as well as ornamental. It seems to the reviewer that the editors of this volume have succeeded admirably in their task, and have produced a book which not only gives an insight into the mathematical development of the author, but is also valuable as a source book and reference volume.

Since most of the papers have been in print for some time, only a brief survey of the volume is necessary. The papers have been roughly grouped under various headings, which give an indication of the variety of interests of the author, and his ability to make contributions in divergent fields. The first section is devoted to the Theory of Analytic Functions, and the first paper is, of course, his Paris doctoral thesis, entitled *Essai sur l'étude des fonctions données par leur développement de Taylor* (1892). It contains the introduction of the notion of greatest and least of the limits of a sequence utilized in the theorem now known as the Cauchy-Hadamard theorem for the radius of convergence of a power series, which is then applied to the study of conditions for the existence of a finite number of polar singularities on the circle of convergence. The thesis also includes sufficient conditions that the circle of convergence be a line of singularities. The third part of the thesis is not included. The second paper (1893) in this group, on entire functions, contains extensions of Poincaré's results on the relations between the growth of the coefficients, the distribution of zeros, and the order of entire functions. Of the three remaining papers in this set, mention might be made of the paper (1898) giving the relations of the singularities of the function $h(x)=\sum c_n x^n$ to those of $f(x)=\sum a_n x^n$ and $g(x)=\sum b_n x^n$ when $c_n=a_n b_n$. The last paper of this series, a meditation on the generalization of analytic functions (1912), anticipates the type of study of derivatives of functions of real variables fundamental to the theory of quasi-analytic functions.

The section devoted to the Theory of Numbers is number-theoretic mainly by implication. It centers in two studies of the Riemann zeta function, the first of which (1896) demonstrates that $\zeta(s)$ has no zeros on the line $\Re(s)=1$, and shows that the same methods are applicable to certain Dirichlet series. The second (1928) applies the formula generalizing Parseval's theorem relative to two developments in Dirichlet series to the function $\zeta'(s)/\zeta(s)$. The third paper in this set is the famous paper on the maximum value of a determinant (1893).

The next section headed Real Functions contains a single paper, *Sur les*