ON AN INTEGRAL TEST OF R. W. BRINK FOR THE CONVERGENCE OF SERIES

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1. *Introduction.* The test in question is embodied in the following theorem due to R. W. Brink.*

Let $\sum^\infty u_n$ *be a series of positive terms. Also let* $r(x)$ *be a function such that* (i) $r(n)=r_n=u_{n+1}/u_n$, (ii) $0<\lambda\leqq r(x)\leqq\mu$, (iii) $r'(x)$ *exists and is continuous,* $\int^\infty |r'(x)|dx$ *is convergent. Then the convergence of the integral*

$$\int^\infty e^{\int^x \log r(t)dt}dx$$

is necessary and sufficient for the convergence of the series $\sum^\infty u_n$.

It is the object of this note to show that Brink's theorem can be expressed in a more general form (Theorem 3 below) which leads at once to all the ratio tests for the convergence of series associated with Kummer's test. The ratio tests are thus welded into unity from a point of view somewhat different from that adopted by Pringsheim in his classical paper *Allgemeine Theorie der Divergenz und Convergenz von Reihen mit positiven Gliedern.*†

2. *Connection of Brink's Theorem with the Maclaurin-Cauchy Integral Test.* The problem which confronts us in Brink's theorem is clearly that of setting up an integral $\int^x F(t)dt$ whose behaviour at infinity is reflected by a given series $\sum^\infty u_n$. When $\sum^\infty u_n$ has all but a finite number of terms positive, the method employed to establish the Maclaurin-Cauchy integral test shows that the convergence of $\int^\infty F(x)dx$ is sufficient for that of $\sum^\infty u_n$ if for $n\leqq x\leqq n+1$, $0<u_n\leqq F(x)$, $(n=m, m+1, \cdots)$. Denoting u_{n+1}/u_n by r_n, the condition assumed is that

$$r_{n-1}\cdot r_{n-2}\cdots r_m \leqq \frac{F(x)}{u_m}, \qquad (n\leqq x\leqq n+1),$$

* R. W. Brink, *A new integral test for the convergence and divergence of infinite series*, Transactions of this Society, vol. 19 (1918), p. 188.

† Mathematische Annalen, vol. 35 (1890), pp. 359–372.