It is clear from this theorem that the Cartan-Goursat calculus of alternating forms* can be developed in Hausdorff spaces with Banach coordinates.

In conclusion we note that Theorem 4 continues to hold if the numerically valued form ω is replaced by a form ω with values in a Banach space.

CALIFORNIA INSTITUTE OF TECHNOLOGY

ON A THEOREM OF ENGEL†

BY MAX ZORN

1. *Introduction.* The theorem of Engel which we intend to study in this paper deals with Lie algebras where an identity $(a(a(a\cdots(ab))\cdots))=0$ holds for arbitrary elements a and b. Under various assumptions it has been shown that in this case all products with sufficiently many factors vanish.

The latest result in this direction was a proof,‡ found first by van Kampen, which holds for finite Lie algebras over any field of characterstic zero. The method is *rational*, but it involves the theory of associative algebras and the theory of traces. Another proof of equal generality, with less accent on the theory of traces, has recently been sketched by the writer.§

The new proof to be offered in the present paper dispenses with every apparatus of matrices, traces, and associative systems. It does not presuppose any knowledge about Lie systems. The material advantage of our direct method is the fact that no reference field is required, and that the question of characteristics never enters the discussion.

2. *Definitions.* DEFINITION 1. *A system L of elements $a, b, \cdots$ is called a Lie ring (with respect to a commutative ring* P *of*

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* E. Goursat, *Leçons sur le Problème de Pfaff*, 1922; E. Cartan, *Leçons sur les Invariants Intégraux*, 1922; E. Kahler, *Einführung in die Theorie der Systeme von Differentialgleichungen*, 1934.

† Substituted for another paper, which was presented to the Society, June 18, 1936. See the last footnote on this page.

‡ See N. Jacobson, *Rational methods in the theory of Lie algebras*, Annals of Mathematics, vol. 36, p. 875.

§ See this Bulletin, Abstract 42-7-266. (Erroneously the theorem in question is there attributed to Lie.)