GENERAL TENSOR ANALYSIS*

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1. *Introduction.* The *coordinates* of the traditional tensor analysis are points in a finite dimensional arithmetic space while those of the author's infinitely dimensional tensor analysis are points (functions) in a function space. The purpose of the present paper is to present the elements of a general tensor analysis that will include as instances the tensor calculi just mentioned. This general point of view has already been instrumental in the more elegant and further development of the function space instances.

For a space of *coordinates* we take a Banach space. The geometrical objects studied are contravariant vector fields, linear connections, multilinear forms and their covariant differentials. Non-holonomic geometric objects are not considered here, as I intend to pursue their study elsewhere.

2. *Abstract Coordinate Transformations.* We consider a Hausdorff† topological space T whose neighborhoods are mapped homeomorphically on an open set S of a Banach space E by mapping functions called *coordinate systems.*‡ If two neighborhoods intersect we have two mappings of their intersection on open subsets S_1 and S_2 of S. This establishes a homeomorphism $\bar{x}(x)$, called a *coordinate transformation*, that takes an open set $S_1 \subset S$ into an open set $S_2 \subset S$. As further restrictions we demand that the Fréchet differentials $\bar{x}(x;\ \delta x)$ and $x(\bar{x};\ \delta\bar{x})$ of $\bar{x}(x)$ and its inverse $x(\bar{x})$ exist in S_1 and S_2, respectively. Finally, to deal with the law of transformation of a linear connection and with covariant differentials, we shall assume that the second Fréchet differential $\bar{x}(x;\ \delta_1 x;\ \delta_2 x)$ exists in S_1 continuously in x.

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† For the purposes of the present paper alone it is merely necessary to postulate that T is a Fréchet neighborhood space and that the coordinate systems are reciprocal (1-1) transformations.

‡ All the results of the paper continue to hold if we take the *coordinate systems* to be homeomorphisms of Hausdorff neighborhoods onto open subsets $\Sigma \subset S$, where S is a fixed open set in E and is itself a homeomorphic map of some Hausdorff neighborhood.