NOTE ON THE IDEALS OF CYCLIC ALGEBRAS*

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1. *Introduction.* The purpose of this note is the generalization of certain results in a recent paper by Latimer† on the number of ideals of given norm in a generalized quaternion algebra.

We consider rational cyclic division algebras D of prime degree $n(\geqq 2)$ over the field R of rational numbers. Let $\mathfrak{o}$ be any maximal order‡ of D. The reduced discriminant of $\mathfrak{o}$ is an invariant $\Delta = \Delta(D)$ of D called the discriminant of D, and is of the form $\Delta = \pm\sigma^{n(n-1)}$, where $\sigma = q_1 \cdots q_s$ is the product of the distinct rational primes $q_1 \cdots q_s$ which are ramified§ in D. For each such q, the two-sided ideal $q\mathfrak{o}$ is the nth power of an indecomposable two-sided prime ideal of $\mathfrak{o}$, and the q-adic extension D_q is a division algebra of degree n of R_q. For all other rational primes p, D_p is the algebra of all matrices of degree n over R_p and $\mathfrak{o}p$ is a two-sided prime ideal of $\mathfrak{o}$ having only one-sided ideal divisors.

By a (normal) ideal of D is meant an ideal (one or two-sided) with respect to some maximal order of D. Such an ideal is called integral if it is contained in its right or left order. We denote various maximal orders by $\mathfrak{o}, \mathfrak{o}_1, \mathfrak{o}_2, \cdots$, and an ideal $\mathfrak{a}$ by $\mathfrak{a}_{ij}$ if $\mathfrak{o}_i\mathfrak{a} = \mathfrak{a}\mathfrak{o}_j = \mathfrak{a}$ and it is necessary to indicate $\mathfrak{o}_i$ and $\mathfrak{o}_j$. The (reduced) norm of an ideal is an ideal of R such that, for a principal ideal $\alpha\mathfrak{o}$ (or $\mathfrak{o}\alpha$), α in D, $N(\alpha\mathfrak{o})$ (or $N(\mathfrak{o}\alpha)) = (N(\alpha))$, where $N(\alpha)$ is the reduced norm corresponding to the rank equation of degree n. Our object is to prove the following result.

THEOREM. *Let $\mathfrak{o}$ be any maximal order of D and let A be any rational integer. Write $A = A_1A_0$, where A_0 is prime to $\Delta(D)$ and every prime factor of A_1 divides $\Delta(D)$. Then the number of integral*

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† C. G. Latimer, Transactions of this Society, vol. 40 (1936), pp. 439–450.

‡ Maximal orders for all D have been given explicitly. See Albert, this Bulletin, vol. 40 (1934), pp. 164–176, for $n=2$, and Hull, Transactions of this Society, vol. 38 (1935), pp. 515–530, for $n>2$.

§ We refer to Deuring, *Algebren*, Ergebnisse der Mathematik, Chapter VI, for all definitions and theorems used here except when explicit reference elsewhere is given.